# MONTHLY NOTICES

OF THE

# ROYAL ASTRONOMICAL SOCIETY

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ROYAL ASTRONOMICAL SOCIETY

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## MEETING OF 1938 NOVEMBER 11

Dr. H. Spencer Jones, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

E. R. Bartlam, M.Eng., M.Inst.Struct.E., F.R.G.S., F.R.S.A., Principal, Ceylon Technical College, Colombo, Ceylon; and

M. H. L. Pryce, M.A., Ph.D., Fellow of Trinity College, Cambridge.

The following candidates were proposed for election as Fellows of the Society, the name of the proposer from personal knowledge being appended :—

Wilfred Atkinson, 2 Duke Street, Penrith, Cumberland (proposed by W. B. Housman);

Abdulazim Aziz, Department of Agriculture, Nicosia, Cyprus (proposed by Thomas L. MacDonald);

Benjamin Bradley, B.Sc., 63 John Street, Brierley Hill, Staffs (proposed by George Lyman);

Gustav Conradie, Lichfield, Stellenbosch, South Africa (proposed by Richard H. Stoy);

Harold Walter Cox, 47 Upper Green, Mitcham, Surrey (proposed by W. M. Lindley);

Henry Rainsford Hulme, M.A., Ph.D., Royal Observatory, Greenwich, London, S.E. 10 (proposed by H. Spencer Jones);

James Gall Inglis, F.R.S.E., 36 Blacket Place, Edinburgh 9 (proposed by E. A. Baker);

George Johnstone, Harbour Master, Braemar, Mostyn, Flintshire (proposed by James Ruthven Brown) ;

Dhundiraj Govind Kale, B.A., 27 Garrick Avenue, London, N.W. 11 (proposed by C. C. L. Gregory) ;

Norman G. Matthew, Secretary, Astronomical Society of Edinburgh, 126 West Savile Terrace, Edinburgh 9 (proposed by W. M. H. Greaves) ;

Thomas Henry Locke Roberts, 105 Oxton Road, Birkenhead (proposed by W. H. Watts) ;

Adolf H. Rosenthal, 32 Kensington Park Gardens, London, W. 11 (proposed by H. Zanstra) ;

Hermann von Socher, Übungs-Sternwarte der Universität, Berlin, N.W. 87 (proposed by Arthur Beer) ;

G. Temple, Ph.D., D.Sc., Professor of Mathematics in the University of London, King's College, Strand, London, W.C. 2 (proposed by A. S. Eddington) ;

William Cameron Walker, M.Sc., Ph.D., The Boulevard Secondary School, Hull, Yorkshire (proposed by F. J. Sellers) ;

Harley Weston Wood, M.Sc., First Assistant Astronomer, Observatory, Sydney, N.S.W., Australia (proposed by James Nangle).

The President announced the deaths of Dr. W. W. Campbell, Associate, and Dr. E. W. Brown, Fellow, and the meeting stood in silence as a tribute to their memory.

The President further announced that the late Dr. E. W. Brown had bequeathed to the Society the sum of five thousand dollars, expressing the desire that the income from this bequest be used for assistance in the calculation of accurate orbits of the bodies of the solar system, such income as shall not be used for this purpose being used for the general purposes of the Society.

One hundred and ninety-six presents were announced as having been received since the last meeting, including :—

E. B. Baillaud, Pour l'Histoire de l'Astronomie Stellaire de Position, 3 vols. (presented by Madame Privat) ;

Portrait of Sir William Herschel, by W. Artaut [the copy painted for Caroline Herschel] (presented by the Rev. Sir John Herschel).

The President drew the attention of the meeting to the above gifts, which are of special value and interest to the Society. The typescript volumes of the late Monsieur E. B. Baillaud represent the results of work done after his retirement from the Paris Observatory, and the typing has been completed by his daughter, Madame Privat, and distributed to those institutions to which her father had intended to send them, and which included the library of the Society.

The portrait of Sir William Herschel, which will replace the inferior copy formerly hanging in the Society's Meeting Room, has been presented by Sir John Herschel to mark the occasion of the bi-centenary of Sir William Herschel's birth on 1938 November 15.

The President announced that a joint meeting with the Physical Society would be held on 1939 January 27, the subject being " The Expanding Universe." The joint meeting would take place in the Physics Theatre at the Imperial College of Science, South Kensington.

Dr. Heber D. Curtis showed some films of moving prominences made at the McMath-Hulbert Observatory of the University of Michigan.

# STAR MODELS WITH VARIABLE POLYTROPIC INDEX.

*Sir A. S. Eddington, O.M., F.R.S.*

1. The polytropic index $n$ at any point in a star is defined by the equation

$$\frac{d}{dr}(\log P) = \left(1 + \frac{1}{n}\right)\frac{d}{dr}(\log \rho). \qquad (1.1)$$

When $n$ is constant this reduces to the familiar formula $P \propto \rho^{1+1/n}$. In this paper we shall investigate some of the general properties of star models in which $n$ increases monotonically from a value $n_1$ at the centre to $n_2$ at the surface.

According to our present knowledge the stars are likely to conform to this specification. From considerations of subatomic energy we expect the source of energy to be rather strongly concentrated to the centre. This tends to make $n$ decrease inwards; but there is a limit to the decrease, because, as soon as $n$ falls below the value corresponding to convective equilibrium, radiative equilibrium gives place to convective equilibrium, and there is a convective core in which $n$ remains steady. The minimum value of $n$ (corresponding to convective monatomic gas) is 1·5. This may be attained in the central core of a star of small mass; for massive stars, in which radiation pressure is large, the minimum is somewhat higher. Thus if we take $n_1 = 1{\cdot}5$, $n_2 = 3{\cdot}5$, or in some cases a rather narrower range, our model should represent ordinary gaseous stars. Rather curiously, the same model with the same range of $n$ is, for quite different reasons, the recognised model for white dwarf stars.

It is useful to introduce for comparison two limiting polytropes $S_1$, $S_2$, which are stars of the same mass and radius with constant polytropic indices $n_1$ and $n_2$ respectively. To focus the inquiry, we may ask to what extent are the properties of the star $S$ with variable index intermediate between those of $S_1$ and $S_2$? One definite result is known: the gravitational potential energy of $S$ is intermediate between the energies of $S_1$ and $S_2$.* Another more technical theorem of this nature has been proved by E. Hopf.† But there has been no general investigation of the extent to which the solution for $S$ lies between the solutions for $S_1$ and $S_2$. The present investigation deals with this problem.

As a rule it is not desirable to combine the study of the deep interior and of the outside layers in one investigation. In practical applications we should therefore take $n_2$ to be the value reached at a point where the usual approximations of the interior begin to break down. The present results then apply on

* *M.N.*, 91, 440, 1931.
† *Zs. f. Ap.*, 3, 71, 1931.

the supposition that the star is completed with $n = n_2$. The effect of possible deviations in the value of $n$ in the outside layers is thus set aside as an independent problem not falling within the scope of the present investigation.

We shall first show that the potential in $S$ satisfies an equation formally similar to Emden's equation, namely,

$$\frac{d^2u}{dz^2} + \frac{2}{z}\frac{du}{dz} + u^\nu = 0, \tag{1.2}$$

where $\nu$ increases continually from $\nu_1$ at the centre to $\nu_2$ at the surface, and

$$n_1 < \nu_1 < \nu_2 < n_2. \tag{1.3}$$

2. Since $n$ is not less than 1·5 in actual stars, we shall assume throughout that $n$ is positive, and in § 4 that it is greater than 1.

By (1.1)

$$P\frac{d\rho}{\rho} = \frac{n}{n+1}dP.$$

Hence

$$d\left(\frac{P}{\rho}\right) = \frac{1}{\rho}\left(dP - \frac{P d\rho}{\rho}\right) = \frac{1}{n+1}\frac{dP}{\rho} = \frac{1}{n+1}d\phi \tag{2.1}$$

by the hydrostatic equation $dP = \rho d\phi$. Integrating inwards from the boundary, with the usual approximation $P/\rho = 0$ at the boundary, we have

$$\frac{P}{\rho} = \frac{1}{n'+1}\phi, \tag{2.2}$$

where $1/(n'+1)$ is a mean value of $1/(n+1)$, so that

$$n < n' < n_2. \tag{2.3}$$

We note that $n'$ decreases inwards. By (2.1) and (2.2)

$$d\left(\log\frac{P}{\rho}\right) = \frac{n'+1}{n+1}d(\log\phi).$$

But by (1.1)

$$d\left(\log\frac{P}{\rho}\right) = d(\log P) - d(\log\rho) = \frac{1}{n}d(\log\rho).$$

Hence

$$d(\log\rho) = \frac{n(n'+1)}{n+1}d(\log\phi) = n''d(\log\phi). \tag{2.4}$$

Having regard to (2.3), we see that

$$n < n'' < n' < n_2, \tag{2.5}$$

and, since $n/(n+1)$ and $(n'+1)$ both increase outwards, $n''$ increases outwards. Integrating (2.4) from the centre outwards, we have

$$\log\rho - \log\rho_c = \nu(\log\phi - \log\phi_c), \tag{2.6}$$

where $\nu$ is a mean value of $n''$, and

$$n_1 < \nu < n'' < n_2. \tag{2.7}$$

We see also that $\nu$ continually increases outwards. The range of $\nu$ (from $\nu_1$ at the centre to $\nu_2$ at the surface) will generally be considerably smaller than the range of $n$. Even when the increase of $n$ is discontinuous and there are stretches of $r$ in which $dn/dr = 0$, the increase of $\nu$ is continuous and $d\nu/dr$ never vanishes.

From (2.6), $\rho/\rho_c = (\phi/\phi_c)^\nu$, so that Poisson's equation gives

$$\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + 4\pi G\rho_c(\phi/\phi_c)^\nu = 0. \tag{2.8}$$

Setting

$$u = \phi/\phi_c, \qquad z = (4\pi G\rho_c/\phi_c)^{\frac{1}{2}}r, \tag{2.85}$$

we reduce (2.8) to the standard form

$$\frac{d^2u}{dz^2} + \frac{2}{z}\frac{du}{dz} + u^\nu = 0, \tag{2.9}$$

with the conditions $u = 1$, $du/dz = 0$, at the centre. This differs from Emden's equation in that $\nu$ is a function of $z$, $d\nu/dz$ being always positive.

3. Fig. 1 shows the general nature of the family of curves constituted by the standard solutions of Emden's equation for different values of $n$. Considering all values of $n$ between $-\infty$ and $+\infty$, the curves completely fill a certain region of the $u$, $z$ plane; and through any point $(z, u)$ in this region there passes one, and only one, curve. We shall call the index of the polytrope which passes through a point $(z, u)$ the *polytropic co-ordinate* of the point.

Near the centre the solution of (2.9) can be found by expansion in series. The first few terms are

$$u = 1 - \tfrac{1}{6}z^2 + \tfrac{1}{120}\nu_1 z^4 + \tfrac{1}{180}\left(\frac{d\nu}{dz}\right)_1 z^5 + \cdots \tag{3.11}$$

$\nu_1$ and $(d\nu/dz)_1$ being the values at the centre. If $p$ is the polytropic co-ordinate, we must have also

$$u = 1 - \tfrac{1}{6}z^2 + \tfrac{1}{120}pz^4 + \text{terms in } z^6. \tag{3.12}$$

Hence initially,

$$p = \nu_1 + \tfrac{2}{3}z\left(\frac{d\nu}{dz}\right)_1. \tag{3.2}$$

To the same approximation $\nu = \nu_1 + z(d\nu/dz)_1$; hence we have initially,

$$\nu_1 < p < \nu. \tag{3.3}$$

If the condition (3.3) continues to be satisfied throughout the whole solution, the solution curve will lie wholly between the two polytropes $\nu_1$ and $\nu_2$, and *a fortiori* between the limiting polytropes $n_1$ and $n_2$. It is therefore important to investigate the possibility of a breakdown of the

condition (3.3) by $p$ becoming equal either to $\nu_1$ or to $\nu$. We shall show that, if a breakdown occurs, it must be caused by $p$ becoming equal to $\nu$; that is to say, our solution cannot cross the $\nu_1$ polytrope unless it has first crossed the $\nu$ polytrope.

For suppose there is a point at which $p$ becomes equal to $\nu_1$. Since $dp/dz$ is initially positive it must have changed sign, and there must be an intermediate point at which $dp/dz = 0$. At this point our solution curve osculates the $p$ polytrope, and has the same value of $u$ and $du/dz$. So long as $dp/dz$ is positive, the solution curve is crossing the polytropes from left to right; it must therefore osculate on the left-hand and lower side of the $p$ polytrope. This means that the curvature $-d^2u/dz^2$ is greater in the

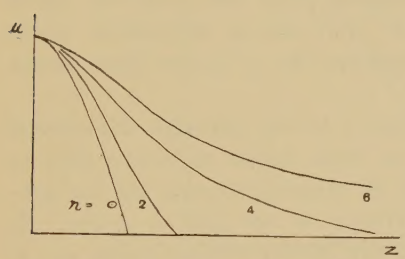


FIG. 1.—*Standard polytropes.*

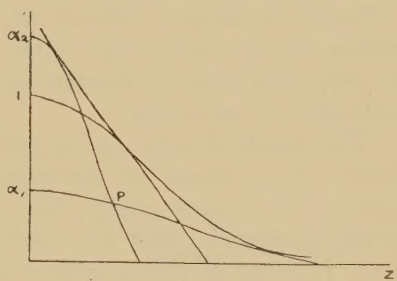


FIG. 2.—*Envelope of the α-family, and α-co-ordinates of a point P.*

solution curve than in the polytrope, and hence by (2.9) $u^\nu > u^p$. Since $u < 1$, this requires $p > \nu$. Thus the osculation, and the subsequent crossing of the $\nu_1$ polytrope, cannot occur until the condition (3.3) has already been broken by $p$ becoming greater than $\nu$.

We shall call the point (if any) where $p$ first becomes equal to $\nu$ the *critical point.* Up to the critical point at least, the solution lies between the two limiting polytropes.

4. If $u = f_n(z)$ is the standard solution of Emden's equation for a constant index $n$, the equation is also satisfied by

$$u = \alpha f_n(\alpha^{\frac{1}{2}(n-1)} z). \tag{4.1}$$

Different values of $\alpha$ give a family of curves which (for $n > 1$) possess an envelope. On the envelope $\partial u/\partial \alpha = 0$, or

$$f_n(\alpha^{\frac{1}{2}(n-1)} z) + \tfrac{1}{2}(n-1) z \alpha^{\frac{1}{2}(n-1)} f_n'(\alpha^{\frac{1}{2}(n-1)} z) = 0. \tag{4.2}$$

Setting $\alpha = 1$ in (4.2), we obtain for the point where the envelope touches the standard polytrope

$$u = -\tfrac{1}{2}(n-1) z u'. \tag{4.3}$$

We call this the *envelope point.*

Through any point P on the left-hand and lower side of the envelope there will pass two curves of the $\alpha$-family (see fig. 2). Let $\alpha_1$, $\alpha_2$ ($\alpha_1 < \alpha_2$) be the corresponding values of $\alpha$. We call $\alpha_1$, $\alpha_2$ the $\alpha$-co-ordinates of P for the index $n$. In particular, for points on our solution curve, we use $\alpha$-co-ordinates corresponding to the index $\nu$ at the point considered.

The solution curve starts (at $z=0$) with $\alpha_1=1$, $\alpha_2=\infty$. As $z$ increases, $p$ becomes less than $\nu$, which means that the solution curve falls below the standard $\nu$ polytrope, and the corresponding $\alpha_1$ co-ordinate is less than 1. We see also that the solution curve crosses the $\alpha_1$ curve from above to below. This direction of crossing can never become reversed. For, if later the crossing were from below to above, there would be an intermediate point at which the solution curve osculates the $\alpha_1$ curve from above. At the osculation point the two curves have the same $u$, $du/dz$ and $\nu$; and therefore by the differential equation they have the same $d^2u/dz^2$. The contact is therefore of the third order. In the solution curve $\nu$ is increasing with $z$; $\nu$ is therefore becoming greater and $u^\nu$ smaller than in the $\alpha_1$ curve; thus $-d^3u/dz^3$ is smaller. Hence at an osculation point the solution curve crosses the $\alpha_1$ curve from below to above; but this is impossible, since we have seen that the solution curve approaches the $\alpha_1$ curves from above until *after* an osculation.

The standard polytrope is one of the two $\alpha$ curves through the critical point. It is crossed by the solution curve from below to above (left to right) at the critical point; this direction of crossing shows, by the preceding paragraph, that the standard polytrope is not the $\alpha_1$ curve. It must therefore be the $\alpha_2$ curve; that is to say, at the critical point $\alpha_2=1$. Since $\alpha_1<\alpha_2$, $\alpha_1<1$. At the envelope point $\alpha_1=\alpha_2=1$; and it is seen from fig. 2 that as $\alpha_1$ decreases, the point moves along the standard polytrope from the envelope point to the surface. We thus reach the result—

If the solution curve crosses the standard $\nu$ polytrope, the crossing takes place *beyond the envelope point.*

5. The following table (based on the tables in Emden's *Gaskugeln*) will give an idea where the envelope point lies:—

| $\nu$ | $u_e$ | $z_e$ | $z_0$ | $M_e/M$ |
|---|---|---|---|---|
| 1·5 | ·216 | 2·81 | 3·66 | ·893 |
| 2·0 | ·364 | 2·54 | 4·35 | ·766 |
| 2·5 | ·463 | 2·35 | 5·42 | ·660 |
| 3·0 | ·541 | 2·16 | 6·90 | ·582 |
| 4·0 | ·641 | 1·92 | 15·00 | ·454 |

The second and third columns give $u$ and $z$ at the envelope point; the fourth column gives the boundary value of $z$; the fifth column gives the proportion of the total mass interior to the envelope point.

It will be seen that, whether reckoned by interior mass or by distance, the envelope point is not unduly far out towards the surface; so that the solution may escape beyond the limiting polytrope in time to make a serious difference to the general properties of the star.

Our investigation has shown that a crossing above the envelope point is impossible, but leaves it uncertain whether a crossing below the envelope point can occur. We can, however, show by an example that the crossing sometimes occurs. In fig. 3 the curve ABCD is the standard $n_2$ polytrope. It is well known that if, starting from D with a smaller gradient $-du/dz$, we integrate the Emden equation inwards, the solution gives a curve of the

form DEBF which goes to $+\infty$ at $z=0$. Referring to fig. 1, we see that one of the standard polytropes must touch DEBF; let AGEH be this polytrope, and let $n_1$ be its index. Then the curve AGED will represent a model with polytropic index $n_1$ in the inner part and $n_2$ in the outer part, the increase occurring discontinuously at E. In this case the critical point

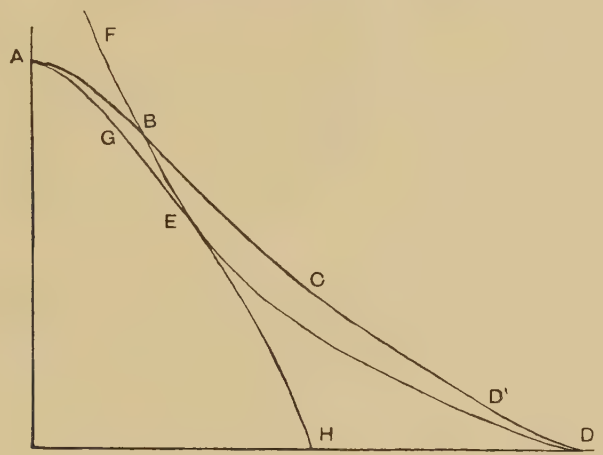


FIG. 3.—*Construction of a composite model AGED.*

D is at the surface; but evidently we could construct in the same way a model with a critical point at D′ by changing the gradient at D′, provided that D′ is not too far inside the surface.

6. Fig. 4 represents a solution curve AECF which crosses the $\nu_2$ polytrope ABCD at C. The star corresponding to it has a relatively more extended

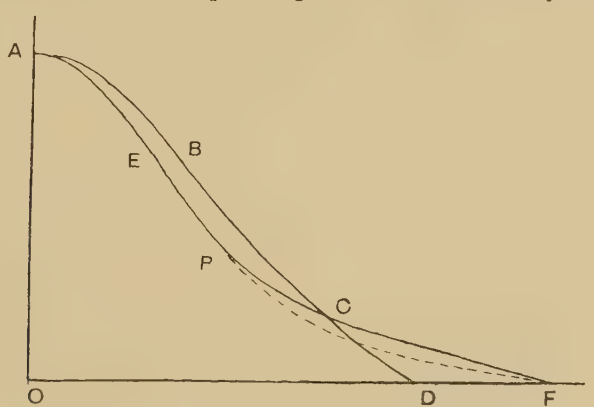


FIG. 4.—*Determination of maximum extension OF.*

envelope than the polytrope; and it is important to investigate the greatest possible amount of extra extension which can be produced by the inward decrease of polytropic index. We therefore consider the following problem.

*Given the limits $\nu_1$ and $\nu_2$, find the condition for the greatest possible extension OF.*

Considering an arbitrary infinitesimal variation $\delta\nu$ of $\nu$ ($\delta\nu$ being a function of $z$) we obtain by differentiating (2.9)

$$\left(\frac{d^2}{dz^2}+\frac{2}{z}\frac{d}{dz}+\nu u^{\nu-1}\right)\delta u = -u^{\nu}\log u\,.\,\delta\nu. \tag{6.1}$$

Starting from the centre, with initial conditions $\delta u = 0$, $d(\delta u)/dz = 0$, we obtain $\delta u$ as a function of $z$ by integrating (6.1). In particular we obtain $\delta u$ at F. If the sign of $\delta\nu$ is reversed throughout, the sign of $\delta u$ at F is reversed. Thus, if a particular variation $\delta\nu$ decreases OF, the opposite variation will increase it. For maximum extension, $\delta u$ at F must be stationary for all possible *reversible* variations of $\nu$. The condition of reversibility is important, because the restrictions placed on $\nu$ may allow increases at certain points but forbid decreases, or *vice versa*.

Consider now a reversible variation $\delta\nu$ limited to a small range $dz$ at a point P, so that (2.9) applies unchanged except at P. At the end of the range there will be an infinitesimal change of gradient $\delta(du/dz)$ of order $\delta\nu\,.\,dz$; the change of $u$ will be an infinitesimal of higher order $\delta\nu\,.\,(dz)^2$, and can be neglected. Thus the new curve leaves AECF at P at a small angle and, to satisfy the stationary condition, crosses it again at F, as shown by the broken curve in fig. 4. Alternatively, the new curve can be traced backwards from F by starting with a value of $du/dz$, which differs infinitesimally from that which gives FCEA, and using the differential equation (2.9) which applies unchanged as far as P. This construction shows that P must be at an intersection of consecutive solutions of (2.9) which start from F.

We have therefore the following necessary (but not always sufficient) condition for the model which gives maximum extension OF : the variation of $\nu$ with $z$ in the model must be such that the restrictions of the problem exclude local reversible changes of $\nu$ except at one or more discrete points where consecutive solutions of the differential equation starting from the surface intersect.

We here take the restrictions to be simply $\nu_1 \leqslant \nu \leqslant \nu_2$, and $d\nu/dz \geqslant 0$. The maximum extension thus obtained will be an upper limit, because in an actual stellar model there are additional restrictions on $\nu$ arising from its construction as a mean value of $n$.

At any point where $d\nu/dz > 0$, infinitesimal variations of $\nu$ of either sign are admissible. We must therefore have $d\nu/dz = 0$, except at the above-mentioned intersection points. If, as seems likely, there is only one intersection point, the required model of maximum extension is a two-phase model consisting of an inner portion with $\nu = \nu_1$ and an outer portion with $\nu = \nu_2$.

If there are two intersection points $z_1$, $z_2$ ($z_1 < z_2$) a three-phase model with three different indices $\nu_1$, $\nu_3$, $\nu_2$ ($\nu_1 < \nu_3 < \nu_2$) is not excluded. But we have then to consider in addition to local reversible changes of $\nu$ at $z_1$ and $z_2$, a change $\delta\nu_3$ applying to the whole range from $z_1$ to $z_2$ which is also reversible in sign. In an outward integration this will produce at $z_2$

a change $\delta(du/dz)$ and also a change $\delta u$ which is no longer an infinitesimal of higher order. (It is conceivable that $\delta u$ might vanish at $z_2$; but this may be dismissed as an unlikely coincidence, since the points $z_1$ and $z_2$ are fixed by an entirely independent condition.) Thus the variations possible at $z_2$ are (*a*) an arbitrary change $\delta(du/dz)$ with no change of $u$, produced by a local change $\delta\nu$ at $z_2$ or at $z_1$, and (*b*) an arbitrary change $\delta(du/dz)$ coupled with a change $\delta u$, produced by a uniform change $\delta\nu$ extending from $z_1$ to $z_2$. It is evident that F cannot be stationary for variations of $du/dz$ at $z_2$ *both with and without* an accompanying change of $u$. We conclude therefore that a three-phase model does not give maximum extension—even if the necessary intersection points exist.

7. The two-phase models introduced in § 6 are not the same as those treated by Milne and others, which consist of two regions of constant $n$ (not constant $\nu$). The present models are very much simpler to calculate, but their application to actual stellar problems is less direct.

But it is possible to extend the method of § 6 to a distribution specified directly in terms of $n$ instead of in terms of $\nu$. We take the restrictions to be as stated in § 1, namely, $n_1 \leqslant n \leqslant n_2$, $dn/dz \geqslant 0$. The latter restriction can be equivalently, and more conveniently, expressed as $-dn/dP \geqslant 0$. Taking fixed values $P_c$, $\rho_c$ of the pressure and density at the centre, we wish to find the model which has maximum radius.

Any one of the possible models can be specified by giving $n$ as a function of $P$. We can then construct the solution by mechanical integration outwards, using (1.1) and the hydrostatic equation. Then, integrating inwards from the surface, starting with an infinitesimally different surface value of $g$, we can determine a consecutive solution. Let $U = \int P d\rho$. If for the two consecutive solutions we plot $U$ as a function of $\rho$, the intersections of the two curves are the points where local reversible variations of $\delta n$ are admissible with the radius remaining stationary. The variables $U$ and $\rho$ are chosen because by (1.1) a local change of $n$ produces a local change of the second derivative $d^2U/d\rho^2$—analogous to the change of $d^2u/dz^2$ in § 6.

It follows as in § 6 that, in the model of maximum radius, $n$ must be constant except for discontinuous increases at the intersection points of consecutive ($U$, $\rho$) curves. Three-phase models are excluded as in § 6, because they introduce two kinds of variation at $z_2$, and the radius cannot be stationary for both types of variation.

It appears therefore that maximum extension is given by a two-phase model with $n = n_1$ in the inner part and $n = n_2$ in the outer part.

The same argument applies also to minimum extension. By § 3 it is not excluded that $p$, after becoming greater than $\nu$, will decrease and ultimately become less than $\nu_1$; but I do not think that this happens. It is probable that the minimum radius is always that of the $\nu_1$ polytrope. Applying the foregoing result, it is sufficient to consider two-phase configurations. If we let the $n_1$ core increase from a mere point until it occupies the whole star, the radius $R$ has initially the value $R_2$ corresponding to the $n_2$ polytrope and finally the value $R_1$ corresponding to the $n_1$ polytrope. My expectation is that intermediately $R$ increases to a maximum and then falls steadily to $R_1$,

which is accordingly the minimum radius. But whether this is so or not should be settled by detailed calculations of two-phase models.

8. Still another modification is needed to enhance the practical value of the investigation. We have investigated the maximum radius of a star of given $P_c$ and $\rho_c$. It would be more useful to determine the maximum radius of a star of given $M$ and $\rho_c$. This is equivalent to determining the maximum central density of a star of given mass and radius.

To introduce an integral property of the star such as $M$, it will be necessary to readjust the scale to the fixed value of $M$ after each variation of $n$. Through this readjustment a local variation of $n$ will cause a change of $U$ as well as of $dU/d\rho$. But I think it is now clear that the same general principles will apply to any extremal problem arising out of our model. The degree of stringency of the stationary condition will remain the same. Applying it to determine the points P, which are such that the stationary condition is satisfied by local reversible variations of $n$ at P, we shall still find only discrete points, though they will no longer be the intersection points of consecutive solutions ; and there will be the same impossibility of satisfying the stationary condition simultaneously both by local variations at P and by uniform variations in a finite range ending at P, except by a highly improbable coincidence. As before, this restricts us to a two-phase model.

We conclude that the extremal value of any selected property of a star, subject to the condition that $n$ increases outwards monotonically from $n_1$ to $n_2$, is given by a two-phase model with an inner portion of constant index $n_1$ and an outer portion of constant index $n_2$. The relative extent of the $n_1$ and $n_2$ portions will depend on the selected property and on the characteristics (e.g. $M$ and $\rho_c$) chosen to be kept constant.

9. *Conclusion.*—We have seen that actual stars are likely to be represented by a model with $n$ continually increasing outwards. Unlimited labour might be spent in performing the integrations on special assumptions as to the law of increase of $n$. But it is more useful to find, if we can, the maximum and minimum values of the stellar quantities we are interested in, instead of particular values corresponding to special assumptions as to the law of increase.

At first it seemed likely that upper and lower limits would be given by the $n_1$ and $n_2$ polytropes. But, so far as we know, the gravitational potential energy is the only quantity to which this applies ; and there are certainly some quantities, *e.g.* the ratio of the central to the mean density, which do not always lie between the values given by the two limiting polytropes.

The actual solution is one stage more complex. We must take the series of two-phase models with a core of index $n_1$ and an exterior shell of index $n_2$, the series being formed by varying the ratio $r_1/R$ of the core radius to the external radius. The extreme values of all quantities will occur in this series. The difference from our original expectation is that the maxima or minima may occur at any value of $r_1/R$, instead of being restricted to $r_1/R = 0$ and 1.

The problem being thus reduced to an investigation of the series of two-phase models, it is quite practicable to carry out the numerical calculations. Probably the most generally interesting extremal problem is to find the

maximum value of $\rho_c/\rho_m$ for indices, say, $1\frac{1}{2}$ and 3. A great amount of numerical work has been done on two-phase models, and it is rather surprising that this has not already been calculated. The result becomes still more important, now that we realise that it will also be the maximum value for *any* law of increase of $n$ from $1\frac{1}{2}$ to 3.

I have not attempted any calculations on these lines, because I think there must already exist a great amount of unpublished calculation of two-phase models, made in other connections, which could be adapted to this purpose.

*Summary.*—The paper is an investigation of the general properties of models in which the polytropic index increases from a value $n_1$ at the centre to $n_2$ at the surface. In §§ 2–5 we examine to what extent the solution for such a model lies wholly between the solutions representing the $n_1$ and $n_2$ polytropes. Since it is found that the solution is not always intermediate, we investigate in §§ 6–8 the problem of finding maximum and minimum values of quantities in which the astronomer is interested, *e.g.* the ratio of the central density to the mean density. It is found that the extremal values are always contained in the series of two-phase models with indices $n_1$, $n_2$. The general significance of the problem and of the results is explained in § 9.

# THE INCLINATIONS OF THE SPIRAL NEBULÆ TO THE LINE OF SIGHT.

*F. G. Brown.*

In a recent analysis * of the extra-galactic nebulæ over 2′ diameter in the Reinmuth catalogue,† the writer found a considerable excess of small ellipticities, the objects with $b/a$ lying between 0 and 1/3 being apparently more than twice as numerous as those in the group 2/3 to 1. On the other hand, Dr. Knox-Shaw ‡ finds from a similar study of the Shapley-Ames catalogue § to the same limiting diameter, approximately equal numbers in each group, and from this contradictory result concludes that the excess of small inclinations is non-existent.

It would appear, however, that this failure to obtain confirmation of the writer's results is due to the automatic exclusion of an undue proportion of spindles over 2′ diameter owing to the integrated magnitudes of many of them being below 13·0. This may be inferred from the internal evidence of the catalogue. The first three lines of Table I record the mean diameters of spirals within certain magnitude limits for each form group, while the final line contains the integrated magnitude $M_t$ of an average spiral of 2′ diameter, assuming a linear relationship between log $d$ and $M_t$ for each group. These figures indicate that only a small proportion of spindles round about 2′ diameter are as bright as the thirteenth magnitude, more than 50 per cent. being below 13·5. On the other hand, comparatively few round spirals of this size are below 13 m.

TABLE I

*Relation between Magnitude, Form and Diameter in the Shapley-Ames Catalogue of Non-Galactic Nebulæ*

| Limits of $b/a$ | 0 to 1/3 | 1/3 to 2/3 | 2/3 to 1·00 |
|---|---|---|---|
| Limits of $M_t$ | Mean Diameter | Mean Diameter | Mean Diameter |
| | ′ | ′ | ′ |
| 10·1 to 11·0 | 9·48 | 7·52 | 4·85 |
| 11·1 to 12·0 | 5·47 | 3·92 | 3·69 |
| 12·1 to 13·0 | 3·21 | 2·59 | 2·05 |
| Mean integrated magnitude at 2′ diameter. | 13·55 | 13·05 | 12·65 |

* *M.N.*, 98, 218, 1938.
† *Heidelberg Veröffentlichungen*, 9, 1926.
‡ *M.N.*, 98, 587, 1938.
§ *Harvard Annals*, 88, 2, 1932.

If this limitation is duly recognised, the Shapley-Ames catalogue furnishes valuable confirmation of the excess of small inclinations, but only for spirals not less than 4′ diameter. The first line of Table II shows that among the larger spirals in the catalogue there are nearly twice as many in the first group as in the third. In the second line are 32 other spirals known to be not less than 4′ diameter but apparently excluded from the catalogue because the integrated magnitude is below 13·0. Of these no less than 25 are narrow spindles, many with the absorption line.

TABLE II

*Frequency of Forms among Spirals not less than 4′ Diameter*

| Limits of *b/a* | 0 to 1/3 | 1/3 to 2/3 | 2/3 to 1·00 |
|---|---|---|---|
| | Number of Objects | | |
| In the Shapley-Ames catalogue | 107 | 102 | 55 |
| Not in the Shapley-Ames catalogue | 25 | 2 | 5 |
| Total | 132 | 104 | 60 |

Below 4′ diameter the proportion of round spirals in the catalogue rapidly increases, so that between 2′ and 3′ there are $2\frac{1}{2}$ times as many objects in the third group than in the first.

In Reinmuth's catalogue the limiting magnitude is generally below 14 m. and consequently, north of $-20°$, it is tolerably complete down to 2′ diameter. Below this limit we find, as in the Shapley-Ames catalogue, a growing excess of round objects.

That the apparent excess of small inclinations does not end here, however, is suggested by a similar study of the small nebulæ in Fath's list of objects found on the Mount Wilson plates of the selected areas * taken with the 60-inch reflector. Table III shows that with the larger scale and much lower limiting magnitude a very marked excess of spindles can be traced down to 30″ diameter, after which the usual "fade-out" begins.

TABLE III

*Frequency of Forms in Fath's List of Nebulæ*

| Limits of *b/a* | 0 to 1/3 | 1/3 to 2/3 | 2/3 to 1·00 |
|---|---|---|---|
| Limits of diameter | Number of Objects | | |
| 47″ and greater | 26 | 12 | 4 |
| 30″ to 46″ | 29 | 23 | 1 |
| 19″ to 29″ | 37 | 123 | 35 |
| 15″ to 18″ | 13 | 22 | 63 |

* *A.J.*, 28, Nos. 10, 11, 1914.

Although there are considerable variations in the mean ellipticity in different parts of the sky, the writer does not find that the excess of spindles is limited to the north galactic polar cap, as suggested by Dr. Knox-Shaw.

*Summary.*—For spiral nebulæ not less than 4′ diameter the Shapley-Ames catalogue confirms the apparent excess of small inclinations found by the writer in the Reinmuth catalogue. Below 4′ the limiting magnitude of 13·0 tends to exclude increasing numbers of spindles relatively to round spirals. The latter catalogue, however, shows that the excess persists down to 2′ diameter, while the small nebulæ in Fath's list confirm its existence at least down to 30″. Whatever its cause, it is not peculiar to any particular catalogue.

# THE STARK EFFECT OF HYDROGEN AND CENTRE-TO-LIMB VARIATIONS IN THE CONTOURS OF $H\beta$, $H\gamma$ AND $H\delta$.

*D. S. Evans, B.A.*

(Communicated by the Director, Cambridge University Observatory)

1. In the laboratory it is found that the lines of the Balmer series of hydrogen are sharp, and that the intensity of successive members decreases rapidly. In stars, however, the lines may be very broad and the intensity decrement small or even reversed.

It has been suggested * that the phenomena of broadening and change of intensity are to be accounted for by the Stark effect.

In the laboratory this is manifested by the splitting of lines in strong electric fields, but in the stellar case the fields responsible are the temporary ones due to the interaction of atoms, and since there is a spread in intensity of these fields the result is a broadening, not a splitting, of the lines, affecting them in a manner analogous to the Doppler effect, but to a much greater extent.

Our picture of the hydrogen lines is thus as follows : We have a narrow core which is broadened by Doppler effect and outside this we have very wide wings which are produced by the Stark effect, which as it were transfers absorption from the centre to the wings. In the present work only these wide wings which make the principal contribution to the equivalent width of the line will be considered. The statistical mechanical theory of the interactions has been given by Holtsmark,† the essential assumption being that for the instant of its operation the temporary field produces the same Stark effect as a static field of the same intensity and direction. To put it classically, the condition for the validity of this assumption is that the interaction field operates for a time which is large compared with the time of orbital revolution in the orbits of relevant quantum number.

Using Holtsmark's results Verweij ‡ has calculated the contours of the Balmer lines $\alpha$, $\beta$, $\gamma$ and $\delta$ for a large number of stars of different temperatures and surface gravity, basing his work on the researches of Pannekoek.§

The object of the present paper is to calculate both the mean contours of $H\beta$, $H\gamma$, and $H\delta$ and the contours which are produced by eccentric zones of an Ao star, in order to determine the variation in intensity of these lines

* A full account is given by Woolley, *Observatory Magazine*, **60**, 225, 1937.

† J. Holtsmark, *Phys. Z.S.*, **25**, 73, 1924.

‡ S. Verweij, "The Stark Effect of Hydrogen in Stellar Spectra," *Publications of the Astronomical Institute of the University of Amsterdam*, No. 5, 1936.

§ A. Pannekoek, "The Theoretical Intensities of Absorption Lines in Stellar Spectra," *Amsterdam Publications*, No. 4, 1935.

from centre to limb and to compare them with the observations of Redman * on the eclipsing binary stars U Cephei and U Sagittæ. He found an intensity decrease towards totality, *i.e.* towards the limb of the primary star, and such a decrease is also found in the present work.

It is easy to see how this comes about. Stark effect provides the hydrogen lines with extensive wings, and at any fixed distance from the centre of the line the effect is roughly proportional to the electron pressure.

Reference to fig. 1 shows that the electron pressure increases almost linearly with optical depth, so that as a rough approximation Stark effect

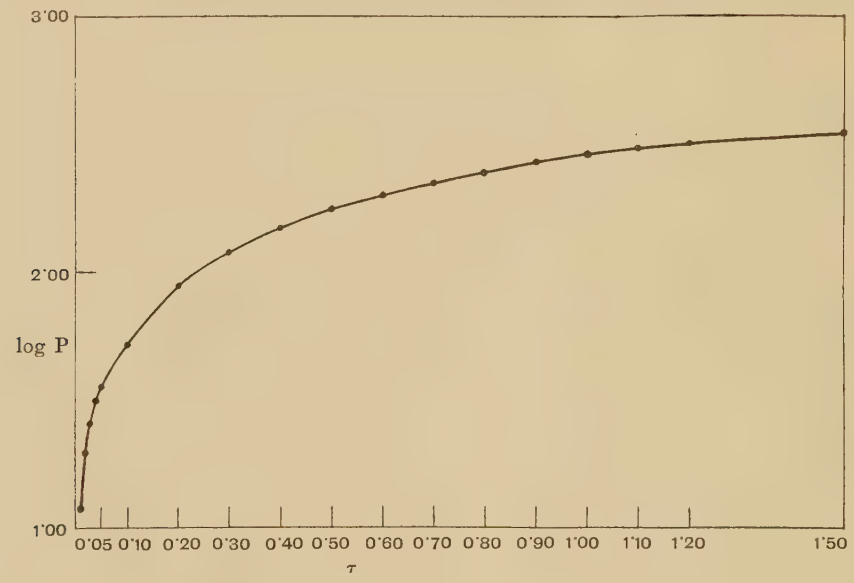


Fig. 1.

increases linearly with optical depth at any fixed wave-length in the wing of the line.

Now a line near the limb of the star is formed at much smaller optical depths than at the centre of the disc, and thus Stark effect is decreased and the line fills up. This, of course, applies only to the wings of the line. It seems probable that the core of the line will not suffer much alteration.

There is, however, another reason why the lines might suffer a decrease in intensity towards the limb. In Eddington's † treatment of the formation of absorption lines a quantity $\epsilon$ is introduced which represents the fraction of energy absorbed which is re-radiated in other frequencies owing to de-excitation by collisions of the second kind (superelastic collisions). In the present work we have taken $\epsilon = 0$. This is in conformity with Verweij's paper, the intention being to follow his treatment as closely as possible, and to study the intensity change deducible from his theory. If we were

* R. O. Redman, "Centre to Limb Variations in Fraunhofer Line Intensity over the Disc of the Primary Star of U Cephei and U Sagittæ," *M.N.*, **96**, 488, 1936.

† *M.N.*, **89**, 620, 1929.

to take $\epsilon = 1$, *i.e.* the case of pure absorption, then the lines would vanish at the limb simply because both line and background would be formed at $\tau = 0$, and the two would be indistinguishable.

The possibility has been suggested * that although de-excitation by collisions is not important the atmosphere may behave as if $\epsilon$ were 1, by the setting up of uncompensated cycles, so that the excited atom might find its way down from the excited state not through a Balmer transition, but through a series of transitions corresponding to other lines of the hydrogen spectrum. In thermodynamical equilibrium this would, of course, be prevented by the operation of detailed balancing.

There appears to be little evidence on which to base an estimate of the effective value of $\epsilon$ in the present case, and, rightly or wrongly, we have followed Verweij in taking $\epsilon = 0$.

2. For the purposes of the present work the important portions of Pannekoek's and Verweij's papers are the tables of the continuous and special absorption coefficients which they give. Pannekoek's work refers to a Russell mixture containing 1000 hydrogen atoms for each metal atom, and he derives, after a detailed consideration of the principal absorption series of all the elements involved, values based on Kramers' theory for the Rosseland mean of the absorption coefficient, $\bar{k}$ (Table 20, p. 39, of his paper), and also values of the continuous absorption coefficient (Table 21) for a wide range of wave-lengths, temperatures and electron pressures.

Verweij, using the Holtsmark formula, gives values of a quantity $s/n$, which is closely related to the special absorption coefficient for each of the lines $H\alpha$, $H\beta$, $H\gamma$ and $H\delta$ for a wide range of the physical parameters and for a long series of values of $\Delta\lambda$—the distance in angstroms from the centres of the lines.

The model star adopted for the work had an effective temperature of about 10,000° ($T_0 = 8400°$ or $5040/T = 0{\cdot}6$) and a surface gravity given by $\log g = 4{\cdot}0$, these values being adopted as a fair representation of the stars studied by Redman.

Throughout the work the optical depth $\tau$ was taken as the independent variable and the set of values used was 0·01, 0·02 up to 0·05 ; 0·10, 0·20, up to 1·20, and 1·50. The first step was, using the adopted value for $T_0$ to calculate by means of Milne's formula :

$$T^4 = T_0^4(1 + \tfrac{3}{2}\tau), \qquad (1)$$

the set of values of $T$ for each of the optical depths given above, and then to construct from Pannekoek's results a series of curves giving the relation between the logarithm of the electron pressure $P$ and the Rosseland mean of the absoprtion cœfficient $\bar{k}$ for each optical depth (temperature) considered.

It was then necessary to solve the hydrostatic equation of support of the atmosphere

$$\bar{k}dp = gd\tau, \qquad (2)$$

where $p$ is the total gas pressure and $g$ the acceleration due to gravity which

* See Strömgren, *Z. f. Ast.*, **10**, 237, 1935 ; Unsöld, *Physik der Sternatmosphären*, p. 288.

is taken constant in the star's atmosphere, and $P$ and $p$ are connected by

$$P = \frac{\bar{x}}{1+\bar{x}} p, \tag{3}$$

where $\bar{x}$ is the mean ionisation, which, since all the metallic atoms are singly ionised at the temperatures considered, is connected with the ionisation $x$ of hydrogen by the relation

$$\bar{x} = x + 0{\cdot}001. \tag{4}$$

Further, we have the Saha formula for the ionisation of the hydrogen given by

$$\log \frac{Px}{1-x} = -\frac{5040}{T} I + \tfrac{5}{2} \log T - 0{\cdot}50, \tag{5}$$

where $I$ is the ionisation potential of hydrogen ($=13{\cdot}54$ volts) and $P$ is in dynes. Second ionisation of the metals is neglected.

(2) can now be written in the form

$$p = \int_0^\tau (g/\bar{k}) d\tau, \tag{6}$$

or

$$\Delta p = (g/\bar{k}) \Delta\tau. \tag{7}$$

If now we calculate (5) for each optical depth, we can solve equation (7) for all depths with the aid of (3), (4) and the set of curves referred to above. The method is as follows: Suppose the equation solved down to a certain depth. Then estimate $\log P$ for the next step in optical depth. In combination with (5) this leads to $x$ and $\bar{x}$. Using the appropriate curve, a knowledge of $\log P$ gives a corresponding value of $\bar{k}$, giving from (7) a value of $\Delta p$ which, added on to the value of $p$ for the previous level, gives $p$ for the new level. Using (3) and the value of $\bar{x}$ already found, we obtain a value of $\log P$ which for consistency should be the same as our estimate. If it is not, a new estimate must be made and the process repeated until consistency is reached, and the next step in optical depth is then attacked. The curve of $\log P$ against $\tau$ so derived is given in fig. 1. Turning to Table 21 of Pannekoek's paper we can now derive by interpolation for each of the lines $H\beta$, $H\gamma$ and $H\delta$ the value of $\log k$ for each optical depth. Table I summarises the first stage of the work.

3. It is now necessary to derive values of the special absorption coefficient for each of the three lines, and for this we need the quantity $\rho(=2P/T)$ which is tabulated above. In Table 6 of his paper Verweij gives values of a quantity $s/n$ which is connected with $\eta$ by the relation

$$\eta = \frac{l}{k} = \left(\frac{s}{n}\right) \frac{(1-x)}{k} f y_2, \tag{8}$$

where $f$ is the oscillator strength for the line,* and $y_2$ is the Boltzmann function:

$$y_2 = 4 \times 10^{-5040 V/T}, \tag{9}$$

where $V = 10{\cdot}15$.

* The values used are those given by Sugiura, *Journal de Physique*, **8**, 113, 1927.

TABLE I

| $\tau$ | $T$ | log $P$ | log $Px/(1-x)$ | log $k$ | | | $\rho$ |
|---|---|---|---|---|---|---|---|
| | | | | $H\beta$ | $H\gamma$ | $H\delta$ | |
| ·01 | 8428 | 1·08 | 1·24 | 0·25 | 0·03 | $\bar{1}$·95 | 2·82 |
| ·02 | 8463 | 1·30 | 1·27 | 0·33 | 0·12 | 0·04 | 4·68 |
| ·03 | 8494 | 1·41 | 1·30 | 0·37 | 0·17 | 0·09 | 6·03 |
| ·04 | 8523 | 1·50 | 1·33 | 0·40 | 0·21 | 0·14 | 7·41 |
| ·05 | 8555 | 1·55 | 1·35 | 0·44 | 0·24 | 0·17 | 8·32 |
| ·10 | 8700 | 1·72 | 1·52 | 0·56 | 0·36 | 0·26 | 12·0 |
| ·20 | 8971 | 1·95 | 1·78 | 0·77 | 0·59 | 0·50 | 20·0 |
| ·30 | 9219 | 2·08 | 2·02 | 0·95 | 0·73 | 0·66 | 26·3 |
| ·40 | 9448 | 2·18 | 2·23 | 1·10 | 0·87 | 0·80 | 31·6 |
| ·50 | 9663 | 2·25 | 2·41 | 1·19 | 0·97 | 0·89 | 36·3 |
| ·60 | 9863 | 2·30 | 2·58 | 1·28 | 1·07 | 0·97 | 40·7 |
| ·70 | 10060 | 2·35 | 2·73 | 1·36 | 1·16 | 1·06 | 44·7 |
| ·80 | 10230 | 2·39 | 2·86 | 1·40 | 1·23 | 1·13 | 47·9 |
| ·90 | 10400 | 2·43 | 2·99 | 1·44 | 1·30 | 1·18 | 51·3 |
| 1·00 | 10560 | 2·46 | 3·11 | 1·48 | 1·32 | 1·22 | 55·0 |
| 1·10 | 10720 | 2·48 | 3·22 | 1·50 | 1·34 | 1·25 | 56·2 |
| 1·20 | 10860 | 2·50 | 3·32 | 1·52 | 1·35 | 1·26 | 57·5 |
| 1·50 | 11280 | 2·55 | 3·59 | 1·56 | 1·39 | 1·30 | 63·1 |

Verweij gives $s/n$ as a function of $\rho$ and $\Delta\lambda$, the distance in angstroms from the centres of the lines for $H\alpha$, $H\beta$, $H\gamma$ and $H\delta$.

The results of this interpolation are given in Tables II, III and IV.

TABLE II

$\eta : H\beta$

| $\tau$ \ $\Delta\lambda$ | 2·0 | 3·0 | 4·0 | 5·0 | 7·0 | 10·0 | 15·0 | 20·0 |
|---|---|---|---|---|---|---|---|---|
| ·01 | 9·60 | 2·96 | 1·78 | ·907 | ·430 | ·144 | ·0513 | ·0247 |
| ·02 | 17·3 | 5·27 | 3·17 | 1·60 | ·780 | ·260 | ·0938 | ·0449 |
| ·03 | 23·3 | 7·10 | 4·16 | 2·15 | 1·06 | ·353 | ·127 | ·0610 |
| ·04 | 30·0 | 9·10 | 5·47 | 2·75 | 1·36 | ·453 | ·163 | ·0783 |
| ·05 | 33·6 | 10·2 | 6·13 | 3·09 | 1·52 | ·510 | ·184 | ·0880 |
| ·10 | 47·0 | 15·0 | 8·40 | 4·23 | 2·10 | ·700 | ·253 | ·121 |
| ·20 | 71·7 | 24·3 | 12·4 | 6·32 | 3·13 | 1·04 | ·375 | ·180 |
| ·30 | 80·5 | 27·9 | 13·6 | 7·24 | 3·50 | 1·17 | ·421 | ·201 |
| ·40 | 81·0 | 28·1 | 13·5 | 7·30 | 3·43 | 1·16 | ·419 | ·201 |
| ·50 | 87·1 | 30·6 | 14·4 | 7·90 | 3·64 | 1·25 | ·451 | ·216 |
| ·60 | 87·1 | 30·5 | 14·6 | 7·92 | 3·61 | 1·25 | ·450 | ·215 |
| ·70 | 86·0 | 30·6 | 14·6 | 7·82 | 3·53 | 1·22 | ·441 | ·211 |
| ·80 | 89·0 | 31·5 | 15·0 | 8·02 | 3·58 | 1·27 | ·450 | ·215 |
| ·90 | 91·5 | 31·5 | 15·1 | 8·08 | 3·56 | 1·31 | ·450 | ·215 |
| 1·00 | 91·8 | 31·0 | 14·9 | 7·97 | 3·47 | 1·31 | ·439 | ·210 |
| 1·10 | 90·2 | 30·3 | 14·6 | 7·80 | 3·37 | 1·28 | ·428 | ·206 |
| 1·20 | 89·0 | 29·7 | 14·0 | 7·67 | 3·30 | 1·27 | ·420 | ·202 |
| 1·50 | 85·2 | 27·8 | 13·5 | 7·20 | 3·06 | 1·22 | ·391 | ·188 |

Table III

$\eta$ : $H\gamma$

| $\tau$ \ $\Delta\lambda$ | 2·0 | 3·0 | 4·0 | 5·0 | 7·0 | 10·0 | 15·0 | 20·0 |
|---|---|---|---|---|---|---|---|---|
| ·01 | 7·98 | 2·29 | 1·10 | ·718 | ·354 | ·118 | ·0425 | ·0235 |
| ·02 | 14·0 | 4·04 | 1·93 | 1·26 | ·630 | ·210 | ·0751 | ·0419 |
| ·03 | 18·6 | 5·60 | 2·55 | 1·67 | ·835 | ·276 | ·100 | ·0557 |
| ·04 | 23·4 | 7·22 | 3·20 | 2·09 | 1·04 | ·349 | ·125 | ·0698 |
| ·05 | 26·9 | 8·40 | 3·68 | 2·40 | 1·20 | ·401 | ·144 | ·0809 |
| ·10 | 36·9 | 12·7 | 5·64 | 3·38 | 1·46 | ·550 | ·195 | ·110 |
| ·20 | 53·3 | 19·8 | 9·18 | 5·08 | 1·91 | ·778 | ·280 | ·156 |
| ·30 | 69·8 | 25·2 | 11·9 | 6·68 | 2·82 | ·960 | ·346 | ·192 |
| ·40 | 74·1 | 26·1 | 12·4 | 6·90 | 3·14 | ·978 | ·355 | ·196 |
| ·50 | 79·0 | 27·6 | 13·0 | 7·30 | 3·46 | 1·04 | ·369 | ·205 |
| ·60 | 81·2 | 27·2 | 12·6 | 7·20 | 3·34 | 1·03 | ·362 | ·200 |
| ·70 | 80·0 | 26·1 | 11·9 | 6·90 | 3·16 | 1·00 | ·345 | ·192 |
| ·80 | 78·1 | 25·5 | 11·5 | 6·60 | 3·00 | ·965 | ·329 | ·183 |
| ·90 | 75·0 | 24·5 | 10·8 | 6·20 | 2·79 | ·908 | ·306 | ·172 |
| 1·00 | 79·2 | 25·9 | 11·2 | 6·40 | 2·85 | ·940 | ·315 | ·175 |
| 1·10 | 77·9 | 25·3 | 11·0 | 6·22 | 2·78 | ·919 | ·306 | ·171 |
| 1·20 | 78·5 | 25·5 | 11·1 | 6·23 | 2·75 | ·920 | ·306 | ·171 |
| 1·50 | 75·8 | 24·5 | 10·5 | 5·88 | 2·55 | ·870 | ·289 | ·160 |

Table IV

$\eta$ : $H\delta$

| $\tau$ \ $\Delta\lambda$ | 2·0 | 3·0 | 4·0 | 5·0 | 7·0 | 10·0 | 15·0 | 20·0 |
|---|---|---|---|---|---|---|---|---|
| ·01 | 7·08 | 2·09 | 1·11 | ·688 | ·260 | ·114 | ·0416 | ·0208 |
| ·02 | 11·8 | 3·70 | 1·95 | 1·22 | ·465 | ·185 | ·0740 | ·0370 |
| ·03 | 16·0 | 5·25 | 2·59 | 1·61 | ·612 | ·269 | ·0980 | ·0491 |
| ·04 | 19·9 | 6·70 | 3·18 | 1·98 | ·755 | ·331 | ·121 | ·0602 |
| ·05 | 23·0 | 7·70 | 3·63 | 2·26 | ·862 | ·380 | ·138 | ·0690 |
| ·10 | 35·9 | 12·2 | 5·70 | 3·31 | 1·27 | ·560 | ·204 | ·102 |
| ·20 | 55·2 | 17·8 | 8·50 | 4·50 | 1·83 | ·776 | ·282 | ·141 |
| ·30 | 72·2 | 21·1 | 10·1 | 5·14 | 2·31 | ·906 | ·331 | ·166 |
| ·40 | 75·8 | 22·0 | 9·95 | 5·20 | 2·40 | ·910 | ·331 | ·166 |
| ·50 | 85·2 | 24·7 | 10·8 | 5·83 | 2·68 | 1·00 | ·361 | ·181 |
| ·60 | 89·1 | 25·9 | 11·2 | 6·16 | 2·73 | 1·02 | ·371 | ·186 |
| ·70 | 87·0 | 25·5 | 11·0 | 6·00 | 2·60 | ·972 | ·355 | ·178 |
| ·80 | 83·2 | 25·0 | 10·6 | 5·80 | 2·48 | ·930 | ·338 | ·168 |
| ·90 | 80·8 | 25·1 | 10·7 | 5·78 | 2·43 | ·914 | ·330 | ·166 |
| 1·00 | 79·0 | 25·4 | 10·8 | 5·71 | 2·38 | ·898 | ·324 | ·162 |
| 1·10 | 75·0 | 24·5 | 10·4 | 5·49 | 2·27 | ·860 | ·309 | ·155 |
| 1·20 | 74·9 | 24·6 | 10·4 | 5·50 | 2·26 | ·859 | ·310 | ·155 |
| 1·50 | 69·9 | 23·9 | 10·0 | 5·22 | 2·12 | ·807 | ·290 | ·145 |

It will be noticed that the march of $\eta$ with $\tau$ is similar in every case. A sharp rise is followed by a long flat maximum, finishing with a decrease at deep levels. This decrease may be the result of neglecting 2nd ionisations of the metals, but it turns out that neither this nor the appearance of irregularities in the march of $\eta$ is of importance for the subsequent work.

4. *Method of Deriving the Residual Intensities.*—The standard equations for Radiative Equilibrium * admit of a finite solution in the following cases :—

$$\text{(i)}\quad \eta = \text{constant},$$

$$\text{(ii)}\quad \frac{1}{1+\eta} = \alpha + \beta\tau,$$

where $\alpha$ and $\beta$ are constants.

It was found that by combining these two solutions the residual intensity for the so-called mean contour could be found with a high level of internal consistency between different approximations.

The following mathematical apparatus was used:—

Let

$$\left.\begin{aligned} \frac{1}{1+\eta} &= \alpha + \beta\tau \qquad (\tau \leqslant \tau_0), \\ \eta &= \eta_0 \qquad\qquad (\tau \geqslant \tau_0), \end{aligned}\right\} \tag{10}$$

where

$$\frac{1}{1+\eta_0} = \alpha + \beta\tau_0.$$

Then for $\tau \leqslant \tau_0$

$$\mathcal{J}_\nu'(\tau) = a + \tfrac{1}{3}b\beta + b\tau + A_1K_0(x) + B_1I_0(x), \tag{11}$$

where

$$\left.\begin{aligned} x &= \frac{6}{\beta}\sqrt{\left(\frac{\alpha+\beta\tau}{3}\right)}, \quad x_0 = \frac{6}{\beta}\sqrt{\frac{\alpha}{3}} \quad \text{and} \quad x_1 = \frac{6}{\beta q}, \end{aligned}\right\} \tag{12}$$

where

$$q^2 = 3(1+\eta_0),$$

and for $\tau \geqslant \tau_0$

$$\mathcal{J}_\nu'(\tau) = a + b\tau + A_2e^{-q\tau}, \tag{13}$$

where the constants $A_1$, $A_2$ and $B_1$ are determined by the continuity conditions

$$\frac{1}{3}\frac{b}{a}\beta + \frac{A_1}{a}(K_0(x_1) - K_1(x_1)) + \frac{B_1}{a}(I_0(x_1) + I_1(x_1)) = 0, \tag{14}$$

$$1 + \frac{b}{a}\left(\frac{\beta}{3} - \frac{2\alpha}{3}\right) + \frac{A_1}{a}\left(K_0(x_0) + 2\sqrt{\frac{\alpha}{3}}K_1(x_0)\right) + \frac{\beta_1}{a}\left(I_0(x_0) - 2\sqrt{\frac{\alpha}{3}}I_1(x_0)\right) = 0, \tag{15}$$

$$\frac{1}{3}\frac{b}{a}\beta + \frac{A_1}{a}K_0(x_1) + \frac{\beta_1}{a}I_0(x_1) = \frac{A_2}{a}e^{-q\tau_0}, \tag{16}$$

* Eddington, *loc. cit.*

where $K_0$, $K_1$, $I_0$, $I_1$ are Bessel functions with imaginary arguments, and $\alpha$, $\beta$, $\eta_0$ are to be determined from the approximations used and

$$r=\frac{3+2\sqrt{3}}{b/a+\sqrt{3}}\frac{\alpha}{3}\left\{\frac{b}{a}+\sqrt{\frac{3}{\alpha}}\left(-\frac{A_1}{a}K_1(x_0)+\frac{B_1}{a}I_1(x_0)\right)\right\}, \tag{17}$$

and it is to be remembered that $I_0$ and $K_0$ are even functions while $I_1$ and $K_1$ are odd.

The procedure adopted then was to graph $\frac{1}{1+\eta}$ and to fit two straight lines, one inclined and one horizontal, and to determine from these lines the values of $\alpha$, $\beta$, $\eta_0$ and $\tau_0$. It should be remembered that the extreme outer layers of the star do not play an important part in the formation of the lines, and that the level playing a predominant part in the formation of the line becomes deeper as $\eta$ decreases.

$\beta$ is negative in all the cases considered, and hence $x_0$, $x_1$ and $x$ are all negative. Formulæ (10) to (17) are a trivial extension of those given by Eddington * when the colour-correction given by Woolley † is included. Thus, following Woolley, we have

$$\mathcal{J}=a+b\tau \tag{18}$$

and

$$H=\frac{b+a\sqrt{3}}{3+2\sqrt{3}} \quad \text{with} \quad \frac{b}{a}=\frac{3h\nu}{8kT_0}. \tag{19}$$

In our case we have the following system of values :—

TABLE V

| | $b/a$ | $H$ |
|---|---|---|
| $H\beta$ | 1·315 | 2·12 |
| $H\gamma$ | 1·473 | 2·02 |
| $H\delta$ | 1·559 | 1·97 |

The residual intensities calculated by these methods for the mean contour are set out in Table VI. It is now necessary to calculate the residual intensities in the contours which arise from particular zones of the primary star.

The zones chosen were those defined by $\cos\theta=\frac{1}{2}$, $\cos\theta=\frac{1}{4}$ and $\cos\theta=\frac{1}{8}$, where $\theta$ is the angle between the line of sight and the radius to the zone considered (see fig. 2). The formulæ used were given by Woolley.‡

The efflux $E$ from solid angle $\delta\omega$ is given by

$$E=\frac{\delta\omega}{4\pi}\int_0^{\xi_1}\{\eta\mathcal{J}_\nu'(P)+\mathcal{J}_\tau\}e^{-(1+\eta)\tau(\xi)}k\rho d\xi, \tag{20}$$

where $PQ$ is the line of sight and $PQ=\xi$, and the integration is carried right through the star, so that $\xi_1$ is the value of $\xi$ corresponding to the farther

* *Loc. cit.*
† Woolley, *M.N.*, **92**, 482.
‡ *M.N.*, **98**, 3, 1938.

end of the chord $PQ$. However, it will be found that only small values of $\xi$ will make an appreciable contribution to the integral, and for such layers we may take $\theta =$ constant.

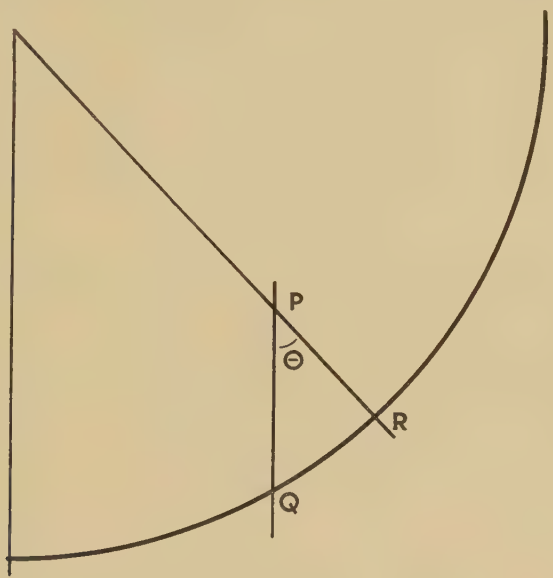


Fig. 2.

Then if $PR$ corresponds to optical depth $\tau$ and $PR = x$

$$d\tau = -k\rho dx.$$

Further,

$$d\tau(\xi) = -k\rho d\xi$$

and

$$\frac{dx}{d\xi} = \cos\theta = \frac{d\tau}{d\tau(\xi)}. \tag{21}$$

Thus we have very nearly

$$\tau = \tau(\xi)\cos\theta, \tag{22}$$

and so (17) becomes

$$E = -\frac{\delta\omega}{4\pi}\int_0^{\tau_1}\{\eta\mathcal{J}_\nu'(\tau) + \mathcal{J}_\tau\}e^{-(1+\tau)\sec\theta}\sec\theta d\tau. \tag{23}$$

Furthermore, $\mathcal{J}_\tau = a + b\tau$ and the continuous emission is given by writing $\eta = 0$ so that the residual intensity is

$$r(\theta) = \frac{\sec\theta\int_0^\infty\{\eta\mathcal{J}_\nu'(\tau) + a + b\tau\}e^{-(1+\eta)\tau\sec\theta}d\tau}{a + b\cos\theta}, \tag{24}$$

where we replace the upper limit of the integral by $\infty$. We shall use this in the form

$$r(\theta) = \frac{\sec\theta\int_0^1\left\{\eta\frac{\mathcal{J}_\nu'(\tau)}{a} + 1 + \frac{b}{a}\tau\right\}e^{-\tau(\sec\theta\sqrt{1+\eta}-1)}d(e^{-\tau})}{1 + b/a\cos\theta}, \tag{25}$$

employing in the computation a method analogous to that used by Eddington,* using $e^{-\tau}$ as independent variable, proceeding in equal steps from 0 to 1 and computing the integrand for each value. The length of step varies according to the rapidity of variation of the integrand, it being necessary to take steps as small as 0·01 in some cases.

The contribution from the layer $\eta =$ constant to $r(\theta)$ may be computed in finite form. It turns out to be

$$\frac{e^{-\tau_0(1+\eta)\sec\theta}}{1+b/a\cos\theta}\left\{1+\frac{b}{a}\tau_0+\frac{b}{a}\frac{\cos\theta}{1+\eta}\right\}+\frac{e^{-\tau_0(q+\sqrt{1+\eta}\sec\theta)}}{(q+\sqrt{1+\eta}\sec\theta)}\frac{\eta\sec\theta}{1+b/a\cos\theta}\frac{A_2}{a}, \quad (26)$$

where all the symbols have already been defined in equations (10) to (17). There appears to be no alternative to numerical integration for the layer where $\eta$ varies.

The residual intensities given by these methods and the results given by Verweij for $\log g = 4{\cdot}0$ and $\frac{5040}{T} = 0{\cdot}5$ and 0·6 are given in Table VI.

TABLE VI

| | | $r$ | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\Delta\lambda =$ | 2·0 | 3·0 | 4·0 | 5·0 | 7·0 | 10·0 | 15·0 | 20·0 |
| $H\beta$ | Mean Contour | ·211 | ·293 | ·375 | ·452 | ·586 | ·738 | ·892 | ·935 |
| | $\cos\theta = \frac{1}{2}$ | ·168 | | | ·340 | ·459 | ·666 | ·836 | ·940 |
| | $\cos\theta = \frac{1}{4}$ | ·187 | | | ·430 | ·554 | ·732 | ·863 | ·940 |
| | $\cos\theta = \frac{1}{8}$ | ·249 | | | ·510 | ·660 | ·816 | ·902 | ·950 |
| | Verweij's results : | | | | | | | | |
| | $5040/T = ·5$ | ·288 | | | ·576 | ·692 | ·805 | ·907 | ·950 |
| | $5040/T = ·6$ | ·192 | | | ·413 | ·535 | ·663 | ·811 | ·882 |
| $H\gamma$ | Mean Contour | ·201 | ·293 | ·394 | ·476 | ·608 | ·770 | ·888 | ·935 |
| | $\cos\theta = \frac{1}{2}$ | ·237 | | | ·371 | | ·681 | ·888 | ·935 |
| | $\cos\theta = \frac{1}{4}$ | ·285 | | | ·431 | | ·738 | ·927 | ·962 |
| | $\cos\theta = \frac{1}{8}$ | ·360 | | | ·488 | | ·843 | ·942 | ·975 |
| | Verweij's results : | | | | | | | | |
| | $5040/T = ·5$ | ·258 | | | ·550 | ·676 | ·793 | ·906 | ·951 |
| | $5040/T = ·6$ | ·159 | | | ·382 | ·504 | ·646 | ·796 | ·879 |
| $H\delta$ | Mean Contour | ·205 | ·293 | ·390 | ·480 | ·625 | ·770 | ·890 | ·932 |
| | $\cos\theta = \frac{1}{2}$ | ·188 | | | ·443 | | ·794 | ·892 | ·940 |
| | $\cos\theta = \frac{1}{4}$ | ·220 | | | ·521 | | ·873 | ·930 | ·960 |
| | $\cos\theta = \frac{1}{8}$ | ·270 | | | ·622 | | ·882 | ·947 | ·975 |
| | Verweij's results : | | | | | | | | |
| | $5040/T = ·5$ | ·234 | | | ·545 | ·669 | ·794 | ·907 | ·948 |
| | $5040/T = ·6$ | ·148 | | | ·369 | ·494 | ·646 | ·797 | ·884 |

* *I.C.S.*, p. 327.

Verweij's results apply to a constant temperature atmosphere, and it will be noticed that the values for the mean contour given here pass from approximate equality with Verweij's results for $5040/T = 0{\cdot}6$ at the centre of the lines, where $\eta$ is large and the dominating layers near the star's surface, towards his values for 0·5 in the wings, where the line is formed at much deeper and hotter layers.

Thus, although a method different from Verweij's has been used in the present paper necessitated by the further applications to centre-to-limb variations, the results for the mean contour may be regarded as consistent with his.

The centres of the lines have been avoided since the present discussion neglects effects which are known to play a part in the modification of the central intensity.

It should be noticed that the contours arising from even such extreme zones as $\cos\theta = \frac{1}{8}$ are of some importance in the formation of the mean since their deviations from it are comparatively large, and an estimate of the importance of the limb contours can easily be made by noting that they must balance out the fairly large opposite deviation of $\cos\theta = \frac{1}{2}$.

However, it must be remembered that the so-called mean contour is not strictly a mean formed by weighting each zone with its area and a darkening factor.

The results for *Hβ* are illustrated in figs. 3 and 4.

5. *Comparison with Observation.*—It is now possible to compute the contours which would actually be observed at different phases of the eclipse of the primary star by a dark companion. For this purpose the companion star was taken to have a radius 1·66 times that of the primary,* and the phases chosen were (i) outside totality (mean contour), (ii) primary star obscured on the line of centres up to the point on its disc defined by $\cos\theta = \frac{1}{2}$, and (iii) primary obscured to $\cos\theta = \frac{1}{8}$. In this last case the actual contour for $\cos\theta = \frac{1}{8}$ was taken, for although the true contour may be different, the contour for $\cos\theta = \frac{1}{8}$ must be a fairly good representation of the true contour at a point with a slightly different value of $\cos\theta$, and since $\cos\theta$ is small the error in phase is minute. In any case it would be extremely difficult so near the limb to make an accurate estimate of the influence of an additional, more eccentric minute area. The question of the centrality or non-centrality of the eclipse only has the effect of altering the phase at which the primary is obscured to a given zone. In our case $i = 84°{\cdot}5$.

The calculation is straightforward. The contour $\cos\theta = \frac{1}{2}$ was associated with the portions of the zone from $\cos\theta = \frac{3}{4}$ to $\cos\theta = \frac{3}{8}$, which was unobscured; $\cos\theta = \frac{1}{4}$ with the zone $\cos\theta = \frac{3}{8}$ to $\cos\theta = \frac{3}{16}$, and $\cos\theta = \frac{1}{8}$ with the remainder. The appropriate darkening factor $1 + b/a\cos\theta$ was introduced for each line, and the weighted mean for each distance from the line centre computed.

The results are given in Table VII.

* See Redman, *loc. cit.*

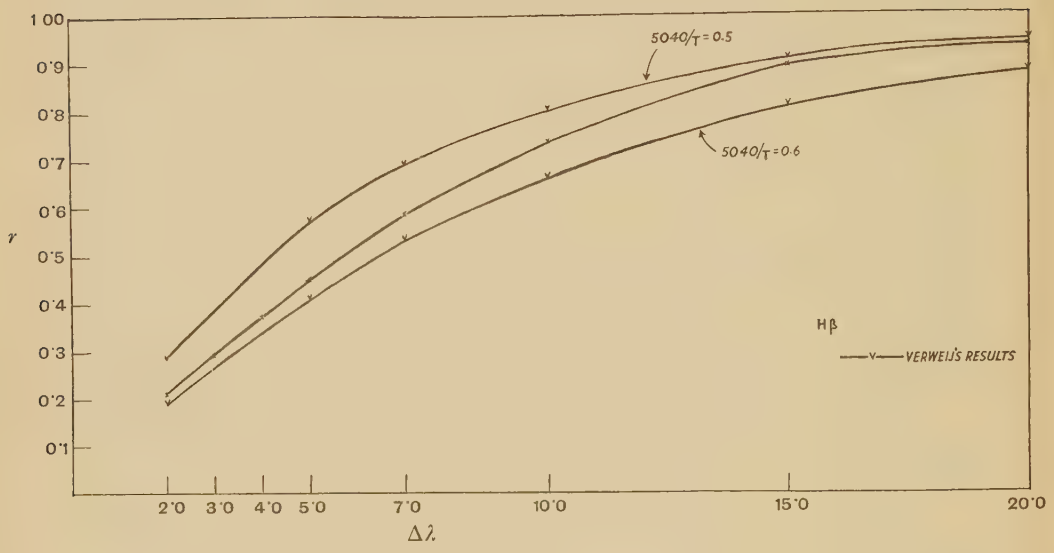
1·00
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
r
5040/T = 0·5
5040/T = 0·6
Hβ
VERWEIJ'S RESULTS
2·0
3·0
4·0
5·0
7·0
10·0
15·0
20·0
Δλ

FIG. 3.

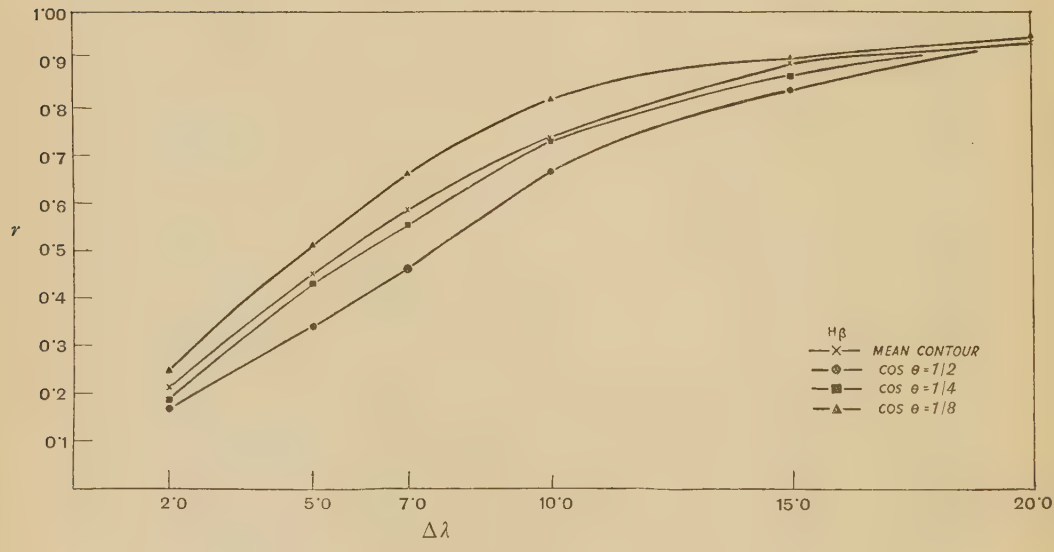
1·00
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
r
Hβ
MEAN CONTOUR
COS θ = 1/2
COS θ = 1/4
COS θ = 1/8
2·0
5·0
7·0
10·0
15·0
20·0
Δλ

FIG. 4.

TABLE VII

| | $r$ | | | | |
|---|---|---|---|---|---|
| $\Delta\lambda =$ | 2·0 | 5·0 | 10·0 | 15·0 | 20·0 |
| *Hβ* Mean | ·211 | ·452 | ·738 | ·892 | ·935 |
| cos $\theta = \frac{1}{2}$ | ·190 | ·415 | ·727 | ·862 | ·945 |
| cos $\theta = \frac{1}{8}$ | ·249 | ·510 | ·816 | ·902 | ·950 |
| *Hγ* Mean | ·201 | ·476 | ·770 | ·888 | ·935 |
| cos $\theta = \frac{1}{2}$ | ·284 | ·420 | ·741 | ·913 | ·952 |
| cos $\theta = \frac{1}{8}$ | ·360 | ·488 | ·843 | ·942 | ·975 |
| *Hδ* Mean | ·205 | ·480 | ·770 | ·890 | ·932 |
| cos $\theta = \frac{1}{2}$ | ·219 | ·511 | ·845 | ·919 | ·955 |
| cos $\theta = \frac{1}{8}$ | ·270 | ·622 | ·882 | ·947 | ·975 |

If now we complete the contours by including a flat centre through the value for $\Delta\lambda = 2$ . A.U. and add exponential wings (the additions are small) we have the following system of equivalent widths.

TABLE VIII

| | *Hβ* | *Hγ* | *Hδ* |
|---|---|---|---|
| Mean | 15·0 | 14·4 | 14·6 |
| cos $\theta = \frac{1}{2}$ | 15·1 | 13·9 | 12·5 |
| cos $\theta = \frac{1}{8}$ | 12·6 | 11·0 | 10·2 |

The inclination $i = 84°{\cdot}5$ places the values for cos $\theta = \frac{1}{2}$ and cos $\theta = \frac{1}{8}$ at stages in the eclipse respectively ·91 and ·998 of the way from 1st or 4th contact to totality.

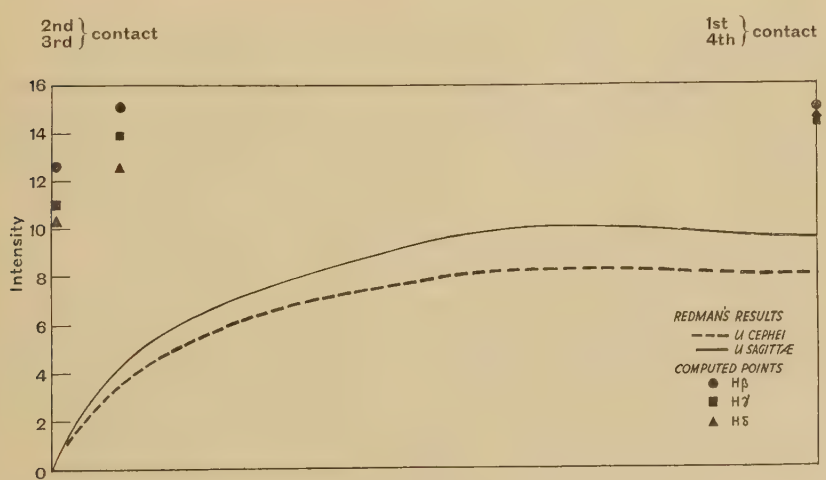


FIG 5.

The results so found are illustrated in fig. 5, together with the curves given by Redman for U Cephei and U Sagittæ. In the figure Redman's results for the mean total line intensity of *Hβ*, *Hγ* and *Hδ* are plotted together with the individual intensities computed in the present paper.

In comparing the results of the computations with the observations, a number of considerations both of a theoretical and a practical nature must be borne in mind.

First, it must be remembered that the lines under consideration are extremely broad, so that even a slight error in fixing the level of the continuous spectrum when measuring the plates would introduce a comparatively large error, amounting perhaps to one or two angstroms in equivalent width and tending to make the observed intensity too low. Secondly, the scatter of the observational results given by Redman is large and tends to make the position of his curves uncertain.

However, there can be no doubt that the values computed here are too high, and so the results given by Verweij have been studied in order to estimate the influence of slight changes in surface gravity and temperature on equivalent width.

The results are given in Table IX. The variation turns out to be large and suggests that measurements of equivalent width of the Balmer lines might be used as a sensitive stellar thermometer.

TABLE IX

*Equivalent Widths (Verweij)*

| | $H\beta$ | | $H\gamma$ | | | $H\delta$ | |
|---|---|---|---|---|---|---|---|
| $5040/T =$ | ·5 | ·6 | ·5 | ·55 | ·6 | ·5 | ·6 |
| $\log g = 4{\cdot}0$ | 12·0 | 20·0 | 12·4 | 15·5 | 18·7 | 12·8 | 18·6 |
| $\log g = 4{\cdot}4$ | 14·4 | 21·5 | 14·9 | 19·1 | 22·5 | 15·5 | 22·5 |

A change from $5040/T = 0{\cdot}6$ to $0{\cdot}5$ may cause a reduction of up to 40 per cent. in equivalent width, while on the other hand the equivalent widths are comparatively insensitive to change in the surface gravity. Thus the fact that there is a displacement of the computed curve away from the observed does not seem to be of great significance and would largely be eliminated by a change from $5040/T = 0{\cdot}6$ to $5040/T = 0{\cdot}5$.

When we come to consider the shape of the curves of variation we notice that the intensity appears to be maintained undecreased in the computed case for a greater proportion of the partial phase than in the observations, and that the final drop of the observed curve is steeper than the computed one.* However, it must be remembered that the last stages of the partial phase yield spectra so faint as to be almost unobservable, and that the fall to zero equivalent width is an assumption.

A point of importance is that in the detailed results set out in Redman's paper, although the point is not certain on account of the scatter of Redman's results, the intensities of $H\gamma$ and $H\delta$, while approximately equal to that of

* In a recent paper K. Walter (*Zeit. f. Ast.*, **16**, 167, 1938) shows that in the case of the U Sagittæ system the dark star has a large ellipticity. By obscuring a greater proportion of the inner zones than a spherical star this would lead to an earlier intensity decrease than that computed here.

*Hβ* outside eclipse, appear to fall off more rapidly than does that of *Hβ*, as is borne out by the computations.

In view of the uncertainties involved in both the observations and the computations, it would seem that the only safe conclusion is that the Pannekoek-Verweij theory predicts qualitatively the effect observed by Redman, but may indicate an intensity decrease which occurs too near totality, while on the other hand it indicates a differential effect between the lines for which there may be some support in the observations.

6. *Summary.*—The derivation of equivalent widths of the Balmer lines by Verweij, taking Stark effect into account, depends on Pannekoek's work, which is based on Kramers' theory. Verweij's theory accounts for the width of the Balmer lines, and the object of the present work is to study the centre-to-limb variations in intensity in order to compare the results with the observations of Redman on the eclipsing binaries U Cephei and U Sagittæ.

The observed intensity decrease is reproduced qualitatively by the Pannekoek-Verweij theory. The computations are based on tables given by Pannekoek and Verweij, and on the formulæ for the formation of lines at the limb given by Woolley.

In conclusion I should like to take this opportunity of expressing my thanks to Dr. Woolley, without whose advice and helpful criticism this paper could not have been written.

*King's College,*
*Cambridge :*
1938 *August.*

*Author's Note.*—Since these computations have been carried out an addendum to *Amsterdam Astron. Inst. Publication*, No. 4, has been published by Pannekoek giving revised values of $k$. In the range considered the revision involves a uniform reduction in $\log \bar{k}$ and $\log k$ of 0·40. A trial computation for *Hβ* has been carried out to estimate the effect of this change.

The results for *Hβ* are :—

| $\Delta\lambda$ | 3·0 | 7·0 | 10·0 | 15·0 | 20·0 | $\left(\frac{5040}{T}=0{\cdot}6 : \log g = 4{\cdot}0\right)$. |
|---|---|---|---|---|---|---|
| $r$ | ·160 | ·360 | ·490 | ·680 | ·785 | |

These values are considerably below those of Table VI. However, it should be remembered that the contours are very sensitive to temperature, and using Verweij's original results as a guide it seems likely that a change from $\frac{5040}{T}=0{\cdot}6$ to 0·5 (a more reasonable value for an A0 star) would produce an increase sufficient to bring the results into fair agreement with the observed contours for α Lyræ given by Miss Williams Elvey and E. G. Williams, which are quoted in Verweij's paper.

Such a change would, of course, also displace the contours originally given by Verweij for a constant temperature atmosphere, and recalculated in the present paper for a variable temperature atmosphere. The observed

contours would then run between these two sets of results. However, in view of our uncertainty as to the exact temperature and surface gravity of the stars concerned, and the sharp dependence of the contour on temperature, the question of agreement with observation ought not to be laboured, since quite a small adjustment of $T$ will secure this. In any case the character of the march of $\eta$ which leads to the decrease of intensity at the limb is preserved.

It should be pointed out that while changes in temperature can secure agreement between theory and observation in this particular case, the high values of the maximum intensity of the Balmer lines still remain and now occur in types earlier than Ao.

This seems to indicate that the Verweij theory gives values of Stark broadening which are too great, but this is not certain since in $\alpha$ Lyræ the Balmer lines are almost twice as intense as in the stars observed by Redman.

# ON THE DENSITY CONDENSATIONS OF CEPHEID VARIABLES.

*Zdeněk Kopal, D.Sc.*

In the paper entitled "On the Pulsations of a Gaseous Star and the Problem of Cepheid Variables" A. S. Eddington * found that if we assume stars to be complete polytropes, specified by $n=3$, then the following relation holds good :

$$P^2\rho_c=\frac{4\cdot 0\pi}{3G(3\gamma-4)},\quad \text{or}\quad P^2\rho_m=\frac{4\cdot 0\pi}{G\xi_1(3\gamma-4)}\left(\frac{d\theta}{d\xi}\right)_1, \tag{1}$$

where $P$ is the period of pulsation, $\rho_c$ and $\rho_m$ the central and the mean density, $G$ the gravitation constant, $\xi_1$ the first zero of the Emden equation, in this case for $n=3$, $\left(\frac{d\theta}{d\xi}\right)_1$ the surface density gradient and $\gamma$ the ratio of specific heats. Its value can be fairly well estimated from the physical theory and apart from minor probable variations with the mass it can be expected constant.

Thus, if the assumptions involved in (1) are true, the period of pulsation is predictable if the density is known, and *vice versa*. Eddington found, however, that the theoretical periods come out systematically too short and expressed the opinion that this may be due to the fact that the cepheids possess lower central condensations than the standard model. The dependance of $P^2\rho$ on the polytropic index has been thoroughly investigated by J. C. P. Miller.† His result was that

$$P^2\rho_m=\frac{3(n+1)\pi}{G\xi_1(3\gamma-4)f(n)}\left(\frac{d\theta}{d\xi}\right)_1, \tag{2}$$

where $n$ is the polytropic index, and

$$f(n)=0\cdot 335+0\cdot 076n-0\cdot 029n^2+\ \ldots$$

The left-hand side of (2) can be determined from observations ; the right side is a function of $n$ and $\gamma$ only. Assuming with Eddington the most probable value of the ratio of specific heats to be $\gamma=\frac{5}{3}$,‡ equation (2) enables us to determine the degree of compressibility of pulsating polytropes from observational data.

It has long been assumed that $P\sqrt{\rho}$ has nearly the same value for all cepheids. The empirical check of M. Güssow § (quoted by Eddington in

* *M.N.*, **79**, 2, 1919.
† *M.N.*, **90**, 59, 1930.
‡ *M.N.*, **92**, 480, 1932.
§ Diss., Berlin, 1927.

his *Internal Constitution of the Stars*) extends to nearly twenty cepheids; the scatter of individual values was large, but there seemed to be no correlation between $P\sqrt{\rho}$ and the period. Further examination of this question was due to Mrs. C. Payne Gaposchkin.* She essentially confirmed the constancy of $P\sqrt{\rho}$, but remarked † that "the scatter of points about the line" (in the $P-\rho$ diagram) "is greater than the errors of observations, and its significance is worth analysing. Evidently one, two or all of the three correlations used in obtaining the mean densities (the period-luminosity law, the period-spectrum relation and the mass-luminosity law) are inexact."

Owing to the interest which the present problem offers in connection with the study of stellar density condensations it seems worth while to rediscuss the question *de novo*. Let us first provide the necessary observational data. The periods of most cepheids are known with a high degree of accuracy, but a source of uncertainty may arise in the determinations of mean densities from the periods and spectra. The procedure is as follows:—

(i) First, to determine the absolute visual, and by applying the appropriate correction, bolometric magnitude $M$, with the aid of the period-luminosity relation.‡

(ii) Secondly, to determine the radius from the temperature and absolute bolometric magnitude. We have:

$$\frac{L_*}{L_\odot}=2{\cdot}512^{M_\odot-M_*},\quad \text{or}\quad \log\frac{L_*}{L_\odot}=\tfrac{2}{5}\Delta M,$$

where

$$\Delta M=M_\odot-M_*.$$

But, on the other hand,

$$\frac{L_*}{L_\odot}=R_*{}^2T_*{}^4.$$

$R$, $T$ expressed in solar units.§ Hence

$$\log R=\tfrac{1}{5}\Delta M-2\log T. \tag{3}$$

(iii) Thirdly, to determine the mass from the mass-luminosity law, and using the computed radius, to find the density. It follows that

$$\log\sqrt{\rho}_m=\tfrac{1}{2}\log\mathfrak{M}-\tfrac{3}{10}\Delta M+3\log T. \tag{4}$$

Eliminating the mass by means of an empirical mass-luminosity law ||

$$\log\mathfrak{M}=0{\cdot}76-0{\cdot}147M+0{\cdot}00655M^2+\ \ldots \tag{5}$$

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* *H.B.*, 876, 1930; *cf.* also C. Payne, "The Stars of High Luminosity," *Harv. Obs. Mon.*, 3, 1930.

† *Loc. cit.*, p. 219.

‡ See Shapley, *Star Clusters*, p. 130, Table X, 1930.

§ The temperatures adopted here are Brill's colour temperatures (*Berl. Bab. Veröff.*, 7, 1). They are somewhat higher than the ionization temperatures, but in the range concerned here the differences are rather unimportant.

|| Z. Kopal, *Zeitschr. f. Astroph.*, 9, 272, 1935.

and inserting $M_{\odot} = +4{\cdot}85$, we finally obtain :

$$\log \sqrt{\rho_m} = -1{\cdot}22 + 0{\cdot}258M + 0{\cdot}0033M^2 + 3 \log T, \qquad (6)$$

where $\rho$ is expressed in terms of the solar density as unit. This equation has been used for calculating the densities.

The following two tables contain the results. They include 88 $\delta$ Cephei stars and 30 short-period variables of the RR Lyræ type. The meaning of individual columns is clear from their headings. Mention need only be made that the spectra given in the third column are median values. All observational data were taken from Schneller's *Katalog und Ephemeriden für* 1938.*

TABLE I

*δ Cephei Stars*

| Star | $P$ | Sp | $\log \frac{T_*}{T_\odot}$ | $-M_{\text{vis.}}$ | $-M_{\text{bol.}}$ | $-\log \sqrt{\rho m}$ | $-\log P\sqrt{\rho m}$ |
|---|---|---|---|---|---|---|---|
| U Aql | 7·024 | G3 | −0·035 | 2·36 | 2·41 | 1·96 | 1·11 |
| SZ Aql | 17·138 | G8 | −0·165 | 3·75 | 4·01 | 2·56 | 1·33 |
| TT Aql | 13·755 | G6·5 | −0·097 | 3·41 | 3·61 | 2·41 | 1·27 |
| $\eta$ Aql | 7·177 | G0·5 | +0·010 | 2·38 | 2·39 | 1·85 | 0·99 |
| Y Aur | 3·859 | F8·5 | +0·043 | 1·55 | 1·56 | 1·48 | 0·89 |
| RT Aur | 3·728 | F8 | +0·054 | 1·52 | 1·52 | 1·43 | 0·86 |
| RX Aur | 11·624 | G5 | −0·070 | 3·13 | 3·24 | 2·19 | 1·13 |
| YZ Aur | 18·193 | G5 | −0·070 | 3·86 | 3·97 | 2·42 | 1·16 |
| RX Cam | 7·192 | F7 | −0·100 | 2·58 | 2·78 | 2·13 | 1·23 |
| U Car | 38·716 | G6·5 | −0·095 | 5·04 | 5·23 | 2·79 | 1·20 |
| V Car | 6·695 | G6 | −0·087 | 2·30 | 2·45 | 2·10 | 1·27 |
| SX Car | 4·860 | G2·5 | −0·023 | 1·86 | 1·86 | 1·77 | 1·09 |
| UX Car | 3·682 | G1 | 0·000 | 1·50 | 1·50 | 1·60 | 1·03 |
| VY Car | 18·984 | G5 | −0·070 | 3·94 | 4·05 | 2·45 | 1·17 |
| *l* Car | 35·520 | G4 | −0·051 | 4·89 | 4·96 | 2·59 | 1·04 |
| SY Aur | 10·145 | G1 | 0·000 | 2·96 | 2·96 | 1·99 | 0·99 |
| RW Cas | 14·801 | K1 | −0·147 | 3·51 | 4·16 | 2·69 | 1·52 |
| RY Cas | 12·136 | G6 | −0·086 | 3·22 | 3·37 | 2·33 | 1·25 |
| SU Cas | 1·949 | F5·5 | +0·105 | 0·92 | 0·93 | 1·15 | 0·86 |
| SW Cas | 5·441 | G5 | −0·070 | 2·03 | 2·14 | 1·98 | 1·24 |
| SY Cas | 4·071 | G1·5 | −0·010 | 1·61 | 1·64 | 1·67 | 1·06 |
| TU Cas | 2·139 | F8·5 | +0·051 | 1·00 | 1·01 | 1·33 | 1·00 |
| XY Cas | 4·502 | G4 | −0·051 | 1·73 | 1·80 | 1·83 | 1·18 |
| V Cen | 5·495 | G1 | 0·000 | 2·05 | 2·05 | 1·75 | 1·01 |
| XX Cen | 10·956 | G2·5 | −0·025 | 3·06 | 3·10 | 2·08 | 1·04 |
| $\delta$ Cep | 5·366 | G0 | +0·012 | 1·97 | 1·98 | 1·66 | 0·96 |
| R Cru | 5·826 | G4 | −0·051 | 2·09 | 2·16 | 1·92 | 1·15 |
| S Cru | 4·690 | G1·5 | −0·004 | 1·80 | 1·81 | 1·69 | 1·02 |
| T Cru | 6·733 | G2·5 | −0·025 | 2·30 | 2·35 | 1·89 | 1·06 |
| X Cru | 6·211 | G5 | −0·070 | 2·17 | 2·28 | 2·01 | 1·22 |
| X Cyg | 16·386 | G4 | −0·051 | 3·67 | 3·75 | 2·41 | 1·20 |
| SU Cyg | 3·846 | F5·5 | +0·110 | 1·55 | 1·57 | 1·29 | 0·70 |
| SZ Cyg | 15·111 | K1 | −0·147 | 3·57 | 4·22 | 2·70 | 1·52 |
| TX Cyg | 14·708 | K4·5 | −0·194 | 3·51 | 4·36 | 2·77 | 1·60 |
| VZ Cyg | 4·865 | G0 | +0·012 | 1·86 | 1·87 | 1·64 | 0·95 |

* *Kl. Veröff. Berl. Babelsberg*, 18, 1937.

TABLE I—*continued*

| Star | $P$ | Sp | $\log \frac{T_*}{T_\odot}$ | $-M_{\text{vis.}}$ | $-M_{\text{bol.}}$ | $-\log \sqrt{\rho m}$ | $-\log P\sqrt{\rho m}$ |
|---|---|---|---|---|---|---|---|
| CD Cyg | 17·071 | G6 | −0·087 | 3·74 | 3·89 | 2·44 | 1·21 |
| $\beta$ Dor | 9·843 | F5·5 | +0·105 | 2·89 | 2·90 | 1·63 | 0·67 |
| $\zeta$ Gem | 10·154 | G1 | 0·000 | 2·96 | 2·96 | 1·99 | 0·98 |
| W Gem | 7·915 | G0·5 | 0·000 | 2·57 | 2·58 | 1·88 | 0·98 |
| RZ Gem | 5·530 | G0 | +0·012 | 2·02 | 2·03 | 1·71 | 0·97 |
| V Lac | 4·984 | F8·5 | +0·051 | 1·87 | 1·89 | 1·55 | 0·85 |
| X Lac | 5·444 | G3 | −0·032 | 2·00 | 2·05 | 1·87 | 1·13 |
| Z Lac | 10·886 | G5 | −0·070 | 3·10 | 3·21 | 2·23 | 1·19 |
| RR Lac | 6·417 | G2·5 | −0·025 | 2·23 | 2·28 | 1·88 | 1·07 |
| BG Lac | 5·332 | G0·5 | +0·005 | 1·97 | 1·99 | 1·73 | 1·00 |
| T Mon | 27·009 | G4 | −0·051 | 4·46 | 4·53 | 2·48 | 1·05 |
| SV Mon | 15·233 | G6·5 | −0·095 | 3·57 | 3·72 | 2·41 | 1·23 |
| R Mus | 7·510 | G1·5 | −0·007 | 2·48 | 2·51 | 1·88 | 1·00 |
| S Mus | 9·661 | G1·5 | −0·007 | 2·89 | 2·92 | 1·98 | 1·00 |
| S Nor | 9·753 | G0 | +0·012 | 2·88 | 2·89 | 1·93 | 0·94 |
| U Nor | 12·643 | G7·5 | −0·106 | 3·28 | 3·50 | 2·41 | 1·31 |
| Y Oph | 17·121 | G2·5 | −0·025 | 3·75 | 3·80 | 2·25 | 1·02 |
| BF Oph | 4·068 | G5 | −0·070 | 1·61 | 1·72 | 1·86 | 1·25 |
| RS Ori | 7·567 | F6 | +0·096 | 2·44 | 2·45 | 1·57 | 0·69 |
| $\kappa$ Pav | 9·106 | G0 | +0·012 | 2·78 | 2·79 | 1·87 | 0·89 |
| SV Per | 11·128 | G4 | −0·051 | 3·09 | 3·17 | 2·17 | 1·12 |
| X Pup | 25·956 | G3·5 | −0·045 | 4·37 | 4·44 | 2·44 | 1·03 |
| RS Pup | 41·337 | K1 | −0·150 | 5·13 | 5·68 | 3·05 | 1·43 |
| S Sge | 8·382 | G2·5 | −0·025 | 2·63 | 2·69 | 1·98 | 1·06 |
| U Sgr | 6·745 | G4 | −0·051 | 2·30 | 2·35 | 1·96 | 1·13 |
| W Sgr | 7·595 | F9 | +0·035 | 2·49 | 2·50 | 1·74 | 0·86 |
| X Sgr | 7·012 | G2 | −0·014 | 2·37 | 2·41 | 1·86 | 1·01 |
| Y Sgr | 5·773 | G1·5 | −0·004 | 2·08 | 2·11 | 1·77 | 1·01 |
| WZ Sgr | 21·850 | G7·5 | −0·106 | 4·14 | 4·36 | 2·59 | 1·25 |
| XX Sgr | 6·424 | G4 | −0·051 | 2·24 | 2·31 | 1·95 | 1·14 |
| YZ Sgr | 9·553 | G7 | −0·100 | 2·85 | 3·05 | 2·28 | 1·30 |
| AP Sgr | 5·058 | G2·5 | −0·025 | 1·91 | 1·95 | 1·80 | 1·09 |
| BB Sgr | 6·637 | G4 | −0·051 | 2·27 | 2·35 | 1·96 | 1·14 |
| RV Sco | 6·061 | G0 | +0·012 | 2·14 | 2·16 | 1·83 | 1·05 |
| RY Sco | 20·314 | G4 | −0·051 | 4·03 | 4·11 | 2·39 | 1·08 |
| X Sct | 4·198 | G3·5 | −0·045 | 1·64 | 1·71 | 1·80 | 1·18 |
| Y Sct | 10·341 | G9·5 | −0·130 | 2·95 | 3·35 | 2·44 | 1·43 |
| Z Sct | 12·902 | K0 | −0·136 | 3·32 | 3·77 | 2·57 | 1·46 |
| RU Sct | 19·696 | K2·5 | −0·165 | 3·98 | 4·70 | 2·87 | 1·58 |
| SS Sct | 3·671 | G1·5 | −0·007 | 1·49 | 1·52 | 1·62 | 1·06 |
| ST Tau | 4·034 | G0 | 0·000 | 1·60 | 1·61 | 1·58 | 0·98 |
| SZ Tau | 3·149 | F8 | +0·067 | 1·34 | 1·35 | 1·37 | 0·87 |
| R TrA | 3·389 | G0 | +0·012 | 1·42 | 1·43 | 1·51 | 0·98 |
| S TrA | 6·324 | G2·5 | −0·025 | 2·20 | 2·24 | 1·86 | 1·06 |
| U TrA | 2·568 | G3·5 | −0·045 | 1·16 | 1·22 | 1·38 | 0·97 |
| T Vel | 4·640 | G2·5 | −0·025 | 1·79 | 1·88 | 1·78 | 1·11 |
| V Vel | 4·371 | G1·5 | −0·007 | 1·70 | 1·88 | 1·73 | 1·09 |
| RY Vel | 28·131 | G7 | −0·100 | 4·54 | 4·74 | 2·68 | 1·23 |
| SV Vel | 14·087 | G3·5 | −0·045 | 3·46 | 3·53 | 2·24 | 1·09 |
| T Vul | 4·436 | F9 | +0·040 | 1·74 | 1·74 | 1·54 | 0·89 |
| U Vul | 7·990 | G4 | −0·051 | 2·56 | 2·64 | 2·04 | 1·14 |
| X Vul | 6·319 | G7·5 | −0·106 | 2·20 | 2·43 | 2·15 | 1·35 |
| SV Vul | 45·172 | K1 | −0·150 | 5·27 | 5·82 | 3·07 | 1·42 |

TABLE II

*RR Lyræ Stars*

| Star | $P$ | Sp | $\log \frac{T_*}{T_\odot}$ | $-M_{bol.}$ | $-\log \sqrt{\rho m}$ | $-\log P\sqrt{\rho m}$ |
|---|---|---|---|---|---|---|
| XX And | 0·108 | A9 | +0·176 | 0·22 | 0·75 | 1·72 |
| SW Aqr | 0·459 | A3 | +0·275 | 0·54 | 0·53 | 0·87 |
| SX Aqr | 0·536 | A2 | +0·313 | 0·66 | 0·45 | 0·72 |
| S Ara | 0·752 | A9 | +0·176 | 0·22 | 0·75 | 1·09 |
| X Ari | 0·651 | A3·5 | +0·271 | 0·49 | 0·53 | 0·72 |
| RS Boo | 0·377 | A8·5 | +0·188 | 0·24 | 0·72 | 1·14 |
| TV Boo | 0·313 | B9 | +0·410 | 1·05 | 0·26 | 0·76 |
| W CVn | 0·552 | F1 | +0·147 | 0·16 | 0·82 | 1·07 |
| Z CVn | 0·654 | A5 | +0·250 | 0·40 | 0·57 | 0·75 |
| RV Cap | 0·448 | A8 | +0·200 | 0·27 | 0·69 | 1·04 |
| RZ Cep | 0·309 | A1·5 | +0·329 | 0·73 | 0·42 | 0·93 |
| RR Cet | 0·553 | F0 | +0·167 | 0·18 | 0·77 | 1·03 |
| XZ Cyg | 0·467 | A4 | +0·263 | 0·46 | 0·55 | 0·88 |
| RW Dra | 0·443 | A5 | +0·250 | 0·40 | 0·57 | 0·92 |
| SU Dra | 0·660 | A3·5 | +0·274 | 0·49 | 0·53 | 0·73 |
| SW Dra | 0·570 | F4 | +0·113 | 0·12 | 0·91 | 1·15 |
| RX Eri | 0·587 | F0 | +0·167 | 0·18 | 0·77 | 1·00 |
| RR Gem | 0·397 | A8 | +0·200 | 0·27 | 0·69 | 1·09 |
| VX Her | 0·455 | A3 | +0·275 | 0·54 | 0·53 | 0·87 |
| RR Leo | 0·452 | A6 | +0·223 | 0·35 | 0·64 | 0·99 |
| U Lep | 0·581 | A4 | +0·268 | 0·47 | 0·54 | 0·78 |
| RR Lyr | 0·567 | A5·5 | +0·235 | 0·38 | 0·61 | 0·86 |
| RZ Lyr | 0·511 | A2 | +0·313 | 0·66 | 0·45 | 0·74 |
| TU Per | 0·607 | A5 | +0·250 | 0·40 | 0·57 | 0·79 |
| RU Psc | 0·390 | A3 | +0·275 | 0·54 | 0·53 | 0·94 |
| V355 Sgr | 0·459 | A2 | +0·313 | 0·66 | 0·45 | 0·79 |
| RU Scl | 0·493 | A0 | +0·375 | 0·91 | 0·33 | 0·64 |
| VY Ser | 0·416 | F5 | +0·102 | 0·10 | 0·99 | 1·37 |
| T Sex | 0·325 | A2 | +0·313 | 0·66 | 0·95 | 0·94 |
| RW TrA | 0·374 | A7 | +0·216 | 0·31 | 0·65 | 1·08 |

The discussion of the above results leads to the following conclusions :—

1. There exists no systematic relation between $P\sqrt{\rho}$ and $P$, but the scatter of points is so large as to make the hypothesis $P\sqrt{\rho}$ to be constant and characteristic for all cepheids scarcely admissible.

2. There exists a remarkable relation between $P\sqrt{\rho}$ and the spectrum, diagramatically shown in the diagram. In view of the inherent uncertainty of observational data the scatter of points for long-period cepheids is strikingly small and strongly suggests that this relation may have a fundamental physical significance. For RR Lyræ stars the scatter is greater, but this may be completely due to larger uncertainties underlying determinations of the densities, viz. the larger bolometric correction, uncertainty of the temperature-scale for earlier types * and the assumption that the absolute visual magnitudes of all short-period cepheids are the same ( $-0^m\cdot10$, according to Shapley.)† The failure of earlier investigators to detect this relation

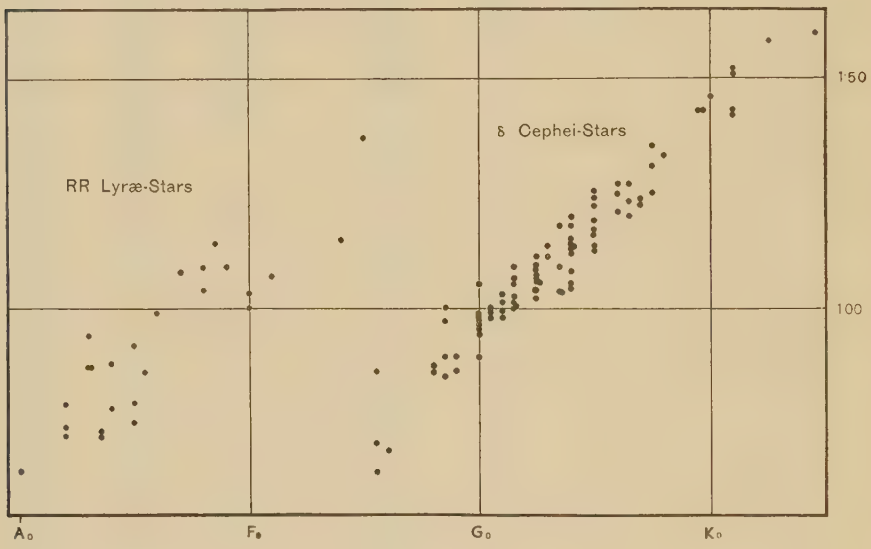
* The temperature scale adopted in the present paper is that of Brill (*Berl. Bab. Veröff.*, **7**, 1).

† See *Star Clusters, loc. cit.*

seems to be due to the fact that they trusted the period-spectrum relation and never tried to plot $P\sqrt{\rho}$ against spectrum.

In view of (2) the physical significance of this result may be stated as follows. *If the hypotheses involved in the present state of the pulsation theory are correct*, the density condensations of cepheids must depend widely on

$$-\log P\sqrt{\rho_m}$$




their spectral class. The stars of the type K–M seem to be highly compressible, nearly as a polytrope $n=4$, but with advancing spectral type the central condensations seem rapidly to diminish, and the δ Cephei-F5 stars seem to approach the limit of homogeneity. The decrease of the density condensation with advancing spectral type seems to apply to short-period cepheids as well.

Another important feature of our diagram is the gap between the long- and short-period cepheids at spectral type F5. Its occurrence at a certain spectral class suggests, however, that the sudden change of $P\sqrt{\rho}$ takes place when a certain temperature has been reached, *i.e.* that the change of the ratio of specific heats might be responsible for it. As long as we take the value of the adiabatic constant to be $\frac{5}{3}$ this explanation is inadmissible because $\gamma$ can vary but slightly, and the one allowance for its possible changes could not explain large changes shown on the diagram. But if $\gamma$ can approach $\frac{4}{3}$, $P\sqrt{\rho}$ becomes exceedingly sensitive to it and the slightest variations might change $P\sqrt{\rho}$ profoundly. Nevertheless, as already mentioned, there is no special likelihood that the ratio of specific heats of a sample of stellar material will be in the neighbourhood of $\frac{4}{3}$, and the only admissible explanation of the gap on our diagram is a real change of the density condensation—which would strongly suggest that the long- and short-period cepheids are of essentially different structure.

*Summary*

Using the period-luminosity and mass-luminosity relations and assuming cepheids to radiate like black bodies, the mean densities have been calculated for 88 δ Cephei stars and 30 variables of the RR Lyræ type. The discussion of the material has led to the following results :—

(i) There is no systematic relation between $P\sqrt{\rho_m}$ and the period.

(ii) There is a well-defined relation between $P\sqrt{\rho_m}$ and the spectrum, which would mean—provided the hypotheses involved in the present state of the pulsation theory of cepheid variables are correct—that the density condensations of cepheids depend widely on their spectral class.

(iii) There is an evident gap in the values $P\sqrt{\rho_m}$ between short- and long-period cepheids, suggesting probably similar gap in the corresponding period-spectrum relation.

In conclusion, it is a great pleasure to express my deep gratitude to Professor Sir A. S. Eddington for stimulating discussions and allowing the very valuable privilege of enjoying the facilities offered by his Observatory where the above work was carried out.

*Cambridge:*
1938 *June.*

# A COMPARISON SEQUENCE FOR NOVA LACERTÆ.

*S. A. Mitchell and C. A. Wirtanen.*

In Stockholm during the sessions of the International Astronomical Union, an informal meeting was held by the leaders of the American, British and French variable star associations. A request was made that the Leander McCormick Observatory should publish the magnitudes and co-ordinates of sequence stars for long-period variables that have been determined since *McCormick Publications*, vol. vi, appeared in 1935. A special request was made for Nova Lacertæ, No. 2; $22^h\ 10^m.0$, $+55°\ 7'$ (1900).

In the appended table are given both the visual and the photovisual magnitudes and also the co-ordinates. Through the generosity of Mount Wilson Observatory and the Carnegie Institution, the McCormick Observatory now possesses a 10-inch Cooke triplet with optical qualities almost identical with the Mount Wilson Cooke camera. With this telescope photographs were obtained of the Nova Lacertæ region and of the north polar sequence taken at the same altitude as the nova. With the 26-inch visual refractor photographs were made through a coarse objective grating. Yellow colour filters, Wratten No. 12, and Eastman I–G Special plates were used with both telescopes. On account of the great speed of this emulsion for photographing in yellow light, the photovisual magnitudes include stars down to 14·2 magnitude. Mr. Wirtanen carried out the measures of the plates by means of the Schilt microphotometer.

With the 26-inch refractor Mr. Mitchell made visual sequences of grade estimates in the manner * followed in the revision of the magnitudes for sequence stars in the 350 long-period variables of the American Association of Variable Star Observers. With zero for the brightest star of the sequence, he plotted the grade estimates against the photovisual magnitudes. As the result of many years' experience, the curve drawn through the grade estimates is always a straight line. In the first column of the table are the magnitudes read off from the straight line. The addition of the visual sequences to the photovisual magnitudes will smooth out the accidental errors. Although the visual magnitudes were obtained to hundredths of a magnitude, they were rounded off to the nearest tenth.

The three faintest stars of the sequence have no photovisual magnitudes. They have been observed independently by Dr. W. H. Steavenson. With the exception of these three stars, a "*d*" chart (scale $20'' = 1$ mm.) has been published by A.A.V.S.O., adopting the magnitudes as given in the first column of the accompanying table.

For various reasons, such as wide double, etc., certain stars were purposely omitted from the published sequence. A total of 30 stars have

* *Publications Leander McCormick Observatory*, **6**, part 2, 1935.

both photovisual and visual magnitudes. Assuming equal accuracy for the two methods, the probable error of one magnitude is 0·02 magnitude. It will be shown in the variable-star publication which will appear within the coming year that the scale of the Mitchell visual magnitudes is in very close agreement with the international photovisual system to which the values of Nova Lacertæ are reduced.

| Magnitudes | | R.A. | Dec. |
|---|---|---|---|
| Visual | Photovisual | | |
| | | m s | ′ |
| 7·3 | | −0 11 | +11·9 |
| 8·5 | 8·45 | −1 21 | − 9·8 |
| 8·8 | 8·78 | +2 20 | −20·3 |
| 9·0 | 9·03 | −0 32 | −10·9 |
| 9·1 | 9·13 | −1 18 | +11·7 |
| 9·2 | 9·21 | +1 1 | − 8·5 |
| 9·5 | 9·50 | +2 34 | − 0·5 |
| 9·7 | 9·71 | +0 15 | −14·0 |
| 10·1 | 10·04 | −0 42 | − 9·1 |
| 10·3 | 10·25 | +0 8 | − 2·1 |
| 10·4 | 10·40 | +0 39 | − 7·1 |
| 10·7 | 10·76 | +1 29 | − 6·9 |
| 11·0 | 11·02 | +1 18 | − 4·3 |
| 11·1 | 11·10 | −0 14 | + 4·6 |
| 11·4 | 11·41 | +0 29 | + 2·2 |
| 11·6 | 11·56 | +0 1 | + 1·1 |
| 11·8 | 11·77 | −0 20 | − 0·6 |
| 12·1 | 11·99 | −0 13 | + 2·8 |
| 12·2 | 12·11 | −0 29 | − 1·8 |
| 12·4 | 12·34 | −0 17 | + 1·2 |
| 12·8 | 12·71 | +0 32 | − 0·9 |
| 13·2 | 13·13 | −0 4 | + 0·9 |
| 13·4 | 13·42 | +0 24 | − 1·6 |
| 13·7 | 13·58 | +0 12 | + 1·3 |
| 14·0 | 13·98 | −0 21 | − 1·5 |
| 14·1 | 14·08 | −0 1 | − 1·9 |
| 14·2 | 14·23 | +0 12 | − 0·7 |
| 14·6 | | +0 3 | + 0·3 |
| 15·0 | | +0 4 | + 0·2 |
| 15·3 | | +0 4 | + 0·4 |

# THE "SOUTHERN STREAM" AND THE $K$ TERM.

*P. Samuels Lall.*

1. Plaskett and Pearce * have offered an explanation for the large $K$ term occurring in B-type stars. The method followed by them has been the subject of strong criticism by Smart.† Malmquist ‡ also has criticised the same. Plaskett § has, however, admitted the error in taking the $K$ term to include the outward effect of the "stream motion" in the equation

$$v_0 \cos D + K = \bar{v}, \tag{1}$$

used by Plaskett and Pearce in determining the "stream motion" of the southern stars of Class B lying between galactic longitudes 216° and 316°, previously investigated by Kapteyn.‖ In his recent paper Plaskett ¶ has given further proof to show that there is a "large outward systematic motion of these southern stars," which, according to him, "certainly shows remarkable community of motion, and as such should be allowed for if reliable values of the motion are to be obtained." Making this allowance, he shows that the $K$ term for these stars is reduced to 1·6 km./sec., which is sufficiently close to the Einstein shift of 1·3 calculated by Plaskett and Pearce, and this, according to him, justifies the "claim that the causes of the high $K$ term in the early solutions of the B-type stars have been found."

There is no objection to using equation (1) for finding the stream motion, *i.e.* the velocity $v_0$ and the convergent point $l_0$, $b_0$, provided that the term $K$ is not taken in a sense other than that of a constant quantity entering, in the nature of an error, in the observed radial velocities. The value of $K$, however, as found from equation (1) by making use of the radial velocities of 132 selected stars occupying a comparatively small region of the sky, would not be of the same value as that determined from a larger number of stars distributed all over the sky. It would be a "local" value of $K$, if one may use this expression. If, however, $K$ were known, the stream convergent could be determined, and the convergent found from proper motions then checked up, as it is generally agreed that "the radial velocity solution may be the more representative of the stellar universe" (Campbell).**

2. It therefore seemed advisable to find the constants of the stream motion from (1) by taking different likely values of $K$, and to use the corresponding position of the convergent to check up the convergent from proper-motion determinations, and thus by a kind of "trial and error" method to arrive at the most probable value of $K$ and position of the convergent.

* *M.N.*, 94, 679, 1934.
† *M.N.*, 96, 568, 1936.
‡ *Handbuch der Astrophysik*, VII, 587, 1936.
§ *M.N.*, 98, 518, 1938.
‖ *Ap. J.*, 40, 43, 1914.
¶ *Loc. cit.*
** *Stellar Motions*, p. 193, 1913.

Transferring $K$ to the right-hand side and putting $\theta = -K$, we get

$$v_0 \cos D = \bar{v} + \theta, \tag{2}$$

where $v_0$ is the stream velocity and $D$ the angular distance of the centre of the group from the vertex of the stream, the position of which is to be determined. $K(=-\theta)$ is the usual $K$ term, and $\bar{v}$ the mean velocity of the corresponding group. The groups taken are the eight groups specified by Plaskett and Pearce in their Table II (data of southern stream).

Making the usual substitutions to change to $x$, $y$, $z$, we get

$$a_1 x + b_1 y + c_1 z = \bar{v} + \theta, \text{ etc.}, \tag{3}$$

where

$$a_1 = \cos \bar{l}_1 \cos \bar{b}_1, \qquad b_1 = \sin \bar{l}_1 \cos \bar{b}_1, \qquad c_1 = \sin \bar{b}_1, \text{ etc.}$$

($\bar{l}_r$, $\bar{b}_r$ are the galactic co-ordinates of the centre of the group area). The quantities $v$, $a$, $b$, etc. are taken (or calculated) from Table II of Plaskett and Pearce, as well as the number of stars in each group, to give the necessary weight to each equation. The values of $x$, $y$, $z$, corresponding to 14 values of $\theta$ ranging from 0 to $-9{\cdot}5$, were then calculated by the method of "least squares," from which the constants $v_0$, $l_0$, $b_0$ for the stream motion were derived. Table I gives the values of $v_0$, $l_0$, $b_0$ for the relevant values of $K(=-\theta)$.

I shall take $C_K$ to represent the *second* position ($l_0 = 214°$, $b_0 = -20°$) of the convergent of the "southern stream" found by Kapteyn from proper motions after a provisional correction in declination; $C_P$ its position ($l_0 = 194°{\cdot}2$, $b_0 = -24°{\cdot}3$) found by Plaskett and Pearce from proper motions; $C_{KP}$ the mean ($l_0 = 204°{\cdot}1$, $b_0 = -22°{\cdot}15$) of the two positions $C_K$ and $C_P$. The position of the convergent found from radial velocities as determined from equations (3) for different values of $K(=-\theta)$ is denoted by $C_R$. In

Table I

| No. | $K=-\theta$ | $v_0$ | $l_0$ | $-b_0$ | $D_K$ | $D_P$ | $D_{KP}$ |
|---|---|---|---|---|---|---|---|
| | | km. | ° | ° | ° | ° | ° |
| 1 | 0 | 18·9 | 225·4 | 8·0 | | | |
| 2 | 1·4 | 17·8 | 221·9 | 9·3 | 13·1 | 30·4 | 21·3 |
| 3′ | 4·0 | 16·0 | 213·9 | 12·0 | 8·0 | | |
| 3 | 4·5 | 15·8 | 212·0 | 12·5 | 7·7 | 20·8 | 14·3 |
| 3″ | 5·5 | 15·1 | 208·5 | 13·7 | 7·9 | | |
| 4′ | 6·5 | 14·6 | 204·7 | 14·8 | | | 7·4 |
| 4 | 7·0 | 14·4 | 202·1 | 15·5 | 11·4 | 11·4 | 6·9 |
| 4″ | 7·5 | 14·2 | 199·6 | 16·0 | | | 8·9 |
| 5′ | 8·5 | 13·9 | 195·4 | 17·1 | | 7·2 | |
| 5 | 9·0 | 13·7 | 193·0 | 17·7 | 33·2 | 6·7 | 14·0 |
| 5″ | 9·5 | 13·6 | 190·7 | 18·2 | | 7·3 | |

Table I, $l_0$, $b_0$ refer to $C_R$, and $D_K$ denotes the distance between $C_K$ and $C_R$ for values of $K$ corresponding to the minimum distance between the two convergents and on either side of it. $D_P$ and $D_{KP}$ represent similar distances

between $C_P$ and $C_R$, and $C_{KP}$ and $C_R$ respectively. The quantities corresponding to the shortest distance between the convergents $C_R$ and those found from proper motions are given in *italics*.

It may be pointed out here that the calculations of Plaskett and Pearce are based on the assumption of the coincidence of the convergents as found from proper motions and from radial velocities. In the investigation in this paper the minimum values of $D_K$, $D_{KP}$, $D_P$ which correspond to

$$K = 4{\cdot}5 \text{ km./sec.}, \qquad K = 7 \text{ km./sec.}, \qquad K = 9 \text{ km./sec.}$$

are found to be 7°·7, 6°·9 and 6°·7 respectively (Table I), and these are considered sufficiently small to justify the remark that the proper-motion convergent fits in fairly well with the corresponding radial-velocity convergent. In examining these results it will be useful to know the mean value of $K$ as found from radial-velocity determinations of the solar-motion constants and the $K$ term. In arriving at this, it was felt desirable to take only those solutions in which there were no assumptions in regard to the position of the solar apex, hence only the five mentioned in Table II have been chosen.

TABLE II

| No. | Solutions | No. | Types | $V_0$ km. | $L_0$ ° | $B_0$ ° | $K$ |
|---|---|---|---|---|---|---|---|
| 1 | Pearce | 680 | O–B5 | 22·0 | 36·8 | 17·3 | 3·5 |
| 2 | Smart | 645 | B | 22·4 | 29·9 | 22·5 | 4·7 |
| 3 | Gyllenberg | 284 | B | 22·1 | 25·4 | 6·7 | 4·3 |
| 4 | Gyllenberg and Malmquist | 240 | B | 23·4 | 35·8 | 12·6 | 4·6 |
| 5 | Nordstrom | 353 | O–B5 | 20·8 | 23·9 | 14·6 | 4·2 |
| 6 | Mean of the above | | | 22·2 | 31·1 | 16·5 | 4·2 |

Weighted means of the solar-motion constants and of $K$ have been calculated and are given in the Table. The value of $K$ thus found comes out as 4·2 km./sec. for the B-type stars. Let us now examine the stream constants given in Table I for different values of $K(=-\theta)$.

3. (i) $K = 0$.—The constants corresponding to $K = 0$ need not be considered, as it is admitted that $K$ must at least be equal to the Einstein shift, which for Class B stars is about 1·4 km./sec. At any rate the convergent for $K = 0$ is too far out from the convergents $C_K$, $C_P$, $C_{KP}$ to be considered as a likely position.

(ii) $K = 1{\cdot}4$.—Taking the mass of B-type stars as 10 times the Sun's mass and their density as 0·1 of the Sun's, the Einstein shift for Class B stars comes out as 1·4 km./sec. According to Plaskett and Pearce, after the outward effect of the stream motion as calculated by them has been allowed for on the whole body of O5 to B7 stars, there remains only a term of 1·6 km./sec., which is "sufficiently close to the gravitational shift of 1·3 km./sec.", their value of the Einstein shift for B-type stars. If we agree that the $K$ term for these stars consists only of the red shift, then the value of the stream constants, as derived from equation (1) by taking $K = 1{\cdot}4$ (which is near enough to both 1·6 and 1·3), ought to lead to a determination of the

convergent, whose distance from the proper-motion convergent should not be very large. We find that for $K = 1{\cdot}4$, $C_R$ is given by

$$l_0 = 221^\circ{\cdot}9, \qquad b_0 = -9^\circ{\cdot}3.$$

The distance of this point from $C_P$ is shown under $D_P$ (Table I) as $30^\circ{\cdot}4$. This obviously makes $C_P$ too far out. The distance $D_K$ between $C_R$ and $C_K$ is $13^\circ{\cdot}1$ and $D_{KP} = 21^\circ{\cdot}3$. We are therefore led to the conclusion that $C_P$ being too far out is an unlikely position of the convergent, and that $C_K$ also is not a good fit. As it is hardly possible that the calculations both of Kapteyn and of Plaskett and Pearce are unreliable, it is concluded that the assumption $K = 1{\cdot}4$ for this class of stars is not correct.

(iii) $K = 4{\cdot}5$.—For the purpose of the present investigation this is sufficiently close to the mean value of $K(=4{\cdot}2)$ found above (Table II) to be considered as the mean value of $K$ for the class of stars now being considered. A glance at Table I shows that the distance $D_K$ between $C_K$ and $C_R$ is minimum for $K = 4{\cdot}5$ and $= 7^\circ{\cdot}7$, whereas $D_P = 20^\circ{\cdot}8$ and $D_{KP} = 14^\circ{\cdot}3$. We are thus led to the conclusion that if the mean value of $K = 4{\cdot}2$ is correct (or nearly correct), the convergents $C_P$ and $C_{KP}$ are both unlikely positions and that Kapteyn's convergent $C_K$ is a very good fit. The corresponding value of $v_0$ is 15·8 km./sec. It might, however, be felt that in taking $K = 4{\cdot}2$ km./sec. (the mean of the five values in Table II) we are accepting too large a value, and consequently the results arrived at are unjustifiable. It may be pointed out that Plaskett,* while advocating a $K$ term of about 1·4 km./sec. for B-type stars as against the larger value, has made a significant remark and is quoted here in full :

"See, however, Trumpler (*Pub. A.S.P.*, 47, 249, 1935), where Einstein shifts of the bright O-type stars in six open clusters are found to average over 9 km./sec."

In view of this result ($K = 9$ km./sec.) for O-type stars as against $K = 2{\cdot}6$ accepted by Plaskett and Pearce (Table V) for the same class of stars, the possibility of the revision of the red shift for B-type stars is not altogether excluded. It may also be added that the question of atmospheric pressure in early B-type stars (suggested by Campbell) † as being one of the possible causes for the large $K$ term for these stars has not been quite abandoned, and there may yet be found other causes in the course of further investigations on stars of this type. The case, therefore, for the value $K = 4{\cdot}2$ km./sec. arrived at by taking the mean of those results in which no assumption has been made in regard to the solar motion is not altogether lost.

It should also be pointed out that the distance between $C_R(K = 4{\cdot}5)$ and the position of the solar vertex ($L = 211^\circ{\cdot}1$, $B = -16^\circ{\cdot}5$) derived from the mean position of the apex (Table II) is only 4·0. That is, the "southern stream" convergent is practically coincident with the solar vertex. It therefore becomes difficult to differentiate the "stream motion" from the solar motion ; this would further strengthen the point raised by Smart ‡ that

* *Loc. cit.*, footnote, p. 520. † *Stellar Motions*, p. 203, 1913.

‡ *Loc. cit.*

the motion detected by Plaskett and Pearce is only the reflex of the solar motion, made conspicuous by the fact of "selected stars," which owing to "their small *motus peculiares* can display the solar motion more clearly than any other type."

"The small *motus peculiaris*" of 5·8 km./sec., obtained by Smart and Green,* has been challenged by Plaskett, who has quoted the unpublished Victoria results from which the random velocity of the B-type stars is found to be 9 km./sec. It should be pointed out that though the velocity of 9 km./sec. for the whole group of Class B stars *does not* exclude the possibility of a lower velocity for the 132 southern stars examined by Plaskett and Pearce for "stream motion," it does, however, make the peculiar velocity of B-type stars very nearly that of the A-type stars for which no such "community of motions" has so far been observed. Hence the matter cannot be settled on this ground until some further light is obtained in regard to the random velocity of *these* 132 southern stars. On the other hand, it has been shown above that if $K$ be taken as 4·2 km./sec., we get a very good fit for $C_K$, but $C_P$ has to be rejected. It is highly unlikely that Plaskett and Pearce's calculations are so far out as to be rejected. Fortunately, we can avoid this difficulty without giving up the mean value of $K(=4{\cdot}2)$ found above for the "whole sky," and come to what may be called a very satisfactory result, in the following manner.

Using equation (1), Plaskett and Pearce have arrived at the value $K=8{\cdot}3$ km./sec. from the motions of the 132 B-type stars which lie in a small region of the sky. This might therefore be taken as Plaskett and Pearce's "*local value*" of $K$ for the region under consideration. As this appeared too large, it was felt that it would be well to get an idea of the "local value" of $K$ from other sources. Hence two solutions for determining $K$ for this region were made by me. Schlesinger's Catalogue of Bright Stars was used, and only stars of the Bo to B5 spectral types were considered. Stars excluded were those with (i) residual velocities greater than $3\frac{1}{2}$ times the average; (ii) doubtful radial velocities. Cluster stars were not excluded. Total number of stars thus considered was 296. Charlier's method of dividing the sky into 48 zones with respect to the galactic plane was followed. The area given by Plaskett and Pearce (in which the 132 "southern stream" stars lie) is covered by 8 of Charlier's southern galactic zones and contains 119 stars. The usual equations, given below, for determining the solar motion by the weighted group method were used:

$$a_1X+b_1Y+cZ+K=\bar{V}_1, \text{ etc.},$$

where $a_1$, $b_1$, etc. have the same meaning as in equation (3) and refer to the 8 zones considered here. Rotational terms were not included, as their effect on the $K$ term for bright B-type stars is known to be practically nil. After allowing the usual weight to each equation, the normal equations were obtained by "least squares," and the value of $K$ obtained therefrom comes out as 6·6 km./sec. The value of $K$ for the same region was also determined from equation (1) by assuming the generally accepted position of the apex

* *M.N.*, **96**, 572, 1936.

($L_0 = 23°{\cdot}5$, $B_0 = 21°{\cdot}6$) and $V_0$ as 22 km./sec. The result was $K = 5{\cdot}6$ km./sec. Taking the mean of the two values, we get the "local value" of $K = 6{\cdot}1$ km./sec. Plaskett and Pearce, using different material and equation (1), have calculated $v_0$ for the "southern stream" and the $K$ term for this region on the *assumption* that the position of the radial-velocity convergent is identically the same as the proper-motion convergent and have arrived at the result $v_0 = 15{\cdot}0$ km./sec. and $K = 8{\cdot}3$ km./sec. The mean of the value (6·1) found by me and of the value (8·3) found by Plaskett and Pearce is 7·2 km./sec. Good confirmation of this result is obtained by the fact that Plaskett * has found the weighted mean value of the residuals of 143 stars of B3 to B7 types in this region as 7·5 km./sec. which is nearly the same as the value arrived at above. It is felt therefore that there is good ground for supposing that for some reason the "local value" of $K(= 7{\cdot}2)$ for this region is much larger than the value 4·2 for the "whole sky." It should also be pointed out that with the "local value" of $K$ at 7·2 km./sec., for Class B stars, the value $K = 4{\cdot}2$ km./sec. for the "whole sky" for the same stars is more probable than the value $K = 1{\cdot}6$ obtained by Plaskett. If we then accept $K = 7{\cdot}2$ km./sec. for this region, we are no longer forced to the conclusion arrived at above of having to reject Plaskett and Pearce's calculation of the convergent of the "southern stream" as determined from proper motions.

(iv) $K = 7$.—The corresponding distances are $D_K = D_P = 11°{\cdot}4$, and $D_{KP} = 6°{\cdot}9$. The distance $D_{KP}$ is minimum when $K = 7$, and the distances $D_K$ and $D_P$ also are not too large; we therefore conclude that $C_{KP}$ is probably the most correct position of the convergent. This result is further confirmed by the fact that the value of $K$ for which $D_{KP}$ is the minimum is practically the same as the "local value" of $K(= 7{\cdot}2)$ found above for this region. Hence on both grounds—that is, on the ground (1) of the minimum distance $D_{KP}$, and (2) of the "local value" of $K$—we are justified in concluding that $C_{KP}(l_0 = 204°{\cdot}1,\ b_0 = -22{\cdot}15)$ is the most likely position of the proper-motion convergent of the "southern stream." $v_0$ comes out $= 14{\cdot}4$ km./sec., which is not very different from Plaskett and Pearce's value (15·0) of the velocity.

It should be mentioned that the solar vertex ($L = 203°{\cdot}5$, $B = -21°{\cdot}6$), as generally accepted, and $C_{KP}$ are coincident (the distance between the two being only 0°·8), and that the distance of $C_R(K = 7)$ from the solar vertex is also small and $= 6°{\cdot}9$. This therefore makes it still more difficult to maintain the differences between the "stream motion" and solar motion. It has, however, been pointed out above that till the question of the actual peculiar velocity of these 132 stars is settled, it is not necessary to identify the two, even though the coincidence of the two positions is very striking. On the other hand, if it is found that the peculiar velocity of these stars is of the size determined by Smart and Green, it would become very difficult to avoid the conclusion that this "community of motion" is the effect of the solar motion on these stars.

(v) $K = 9$.—The distance $D_P$ is minimum and $= 6°{\cdot}7$ when $K = 9$, but $C_K$ and $C_{KP}$ are both too far out. Hence $C_P$ is an acceptable position

* *Loc. cit.*

of the convergent if $K$ be taken as 9 km./sec. This would, however, lead us to reject Kapteyn's convergent as being incorrect, and to accept a value of $K$ even higher than the "local value" found above, both of which are very unlikely. It is also not necessary to entertain these conclusions in virtue of the result arrived at previously (3 (iv)), according to which the results of both Kapteyn, and Plaskett and Pearce, are conserved on the basis of a "local value" of $K$, which is more in accordance with known facts.

4. *Summary.*—(1) The mean value of $K$ has been found from radial-velocity solutions of the solar motion and the $K$ term, in which no assumption has been made in regard to the position of the solar apex, and comes out as 4·2 km./sec. The Einstein shift for B-type stars has been taken as 1·4 km./sec. A "local value" of $K$ has been determined for the "southern-stream" region for B-type stars and found to be 7·2 km./sec.

(2) The equation $V \cos D = \bar{V} - K$ has been used to find $v_0$, $l_0$, $b_0$ corresponding to 14 values of $K$. The convergents $C_R$ corresponding to $K = 1{\cdot}4$, 4·2, 7·2 and 9·0 have been examined with reference to the proper-motion convergents of Kapteyn ($C_K$) and of Plaskett and Pearce ($C_P$), and $C_{KP}$ the mean of the above two.

(3) It is found that $K = 1{\cdot}4$ leads to no correspondence between $C_R$ and the proper-motion convergents. It is suggested, therefore, that $K$ is not of the size 1·4. $K = 4{\cdot}2$ gives a good fit for $C_K$ but not for $C_P$ or $C_{KP}$. It has been shown that it is not necessary to reject $C_P$ if we take $K$ equal to its "local value" for this region, *i.e.* 7·2 km./sec., without sacrificing the value of $K = 4{\cdot}2$ for the whole sky. It is found then that $C_{KP}$ is a very good fit, and that $C_K$ and $C_P$ are not too far out. $K = 9$ leads to a very good fit for $C_P$, but $C_K$ has to be rejected. It is considered unlikely that $C_K$ is altogether wrong and that $K$ is even higher than the "local value" 7·2. Hence $C_{KP}$ is accepted as the correct position of the convergent. It is also more in accord with known facts in regard to the "local value" of $K$. It is pointed out that $C_{KP}$ is *coincident* with the solar vertex, but it is not necessary to identify the "stream" motion with solar motion until further light on the peculiar velocities of the 132 B-type stars forming the "stream" is obtained.

(4) It comes out incidentally as a result of the above conclusions that 4·2 km./sec. is a more probable value of $K$ for the O- to B7-type stars than 1·6 km./sec.

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# STELLAR PARALLAXES DETERMINED AT THE ROYAL OBSERVATORY, GREENWICH.

(Communicated by the Astronomer Royal)

The following list is a continuation of the relative parallaxes obtained with the 26-inch refractor, and published annually in the November number of the *Monthly Notices*.

The visual magnitude and spectral type for stars contained in the *Draper Catalogue* have been extracted from that catalogue. The photographic magnitudes are from the *Greenwich Astrographic Catalogue*.

Two values of the proper motion in R.A. are given. The first (as also the proper motion in Dec.) is absolute, and is based on meridian observations or the comparison of long-interval plates. The second is the relative proper motion derived from the parallax plates.

In the last column the first value of the probable error refers to the parallax, and the second to the relative proper motion in R.A. These are based on the value of the probable error for a plate of unit weight given in the eighth column. The mean probable error for unit weight is $\pm$ 0″·026.

To minimise the effect of *quasi*-seasonal error the observations have been spread over not less than six epochs. The average number of plates per star is 31 ; for those stars observed during the summer months the number is in excess of the average to compensate for the loss of factor.

Generally six comparison stars of mean magnitude 11–12 have been used, the magnitude of the parallax star being reduced to equality by means of a rotating sector. For the brighter stars a neutral-tinted filter absorbing 4·8 magnitudes is used in conjunction with the sector. Two exposures of three minutes each are made on each plate, the carrier being turned through 180° after the first exposure. As the filter is permanently mounted on one of the carriers the reversal should eliminate any distortion due to the filter as well as to film displacement.

Star No. 11 is taken from the list of large proper-motion stars published in *A.J.*, 871 (1926), by Ross, who gives the annual proper motion as 3″·20 at 174°. The Greenwich absolute parallax is +″·039, which agrees closely with the Mount Wilson trigonometric value of +″·042, and corresponds to a visual absolute magnitude of 10·2.

No. 12 is one of a list of test objects recommended by the I.A.U. for observation in *A.J.*, 864 (1926), the purpose being to determine the relative and absolute systematic errors of trigonometric parallaxes. Of these objects 9 are in the Greenwich zone ; the results for 7 have now been published.

Nos. 13 and 18 are both close double stars (see Aitken, *L.O.B.*, 413, 1929) for which dynamical parallaxes of +″·011 and +″·013 respectively have been deduced.

A former determination for No. 21, $\chi$ Draconis, published in the first Greenwich volume of parallaxes (star No. 205), gave $\pi = +''\cdot134 \pm ''\cdot009$. The published results from various observations by trigonometric and other methods being somewhat discordant, the star was re-observed at Greenwich. The new value $\pi = +''\cdot126 \pm ''\cdot010$ is in good agreement with the previous Greenwich observation, but is still somewhat high compared with the others. The star is a spectroscopic binary of period 281$^d$.

Nos. 28 and 29 were measured on the same set of plates, but do not appear to be physically connected.

| No. | Name and B.D. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. 1900·0 | Unit 0″·001 Proper Motion R.A. | Proper Motion Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 1 | R. 28 | 9·0 | Go | 0 8 45 | +165 | − 13 | 19 | − 1 | ± 6 |
| | 85° 2 | 8·6 | | +86° 10′ | +159 | | | | ± 2 |
| 2 | Gr. 67 | 8·4 | Ko | 0 26 12 | + 57 | − 18 | 20 | + 4 | ± 5 |
| | 85° 9 | 8·3 | | +85° 46′ | + 27 | | | | ± 2 |
| 3 | O.A. 847 | 8·8 | Ko | 0 48 42 | + 89 | + 29 | 24 | + 1 | ± 7 |
| | 72° 45 | 9·7 | | +72° 36′ | + 64 | | | | ± 4 |
| 4 | 68° 104 | 9·8 | | 1 19 24 | +153 | −145 | 36 | − 11 | ± 11 |
| | | 9·4 | | +68° 45′ | +139 | | | | ± 5 |
| 5 | L.F. 289–90 | 7·6 | Ko | 1 44 35 | +109 | − 56 | 26 | + 9 | ± 7 |
| | 80° 58 | 8·3 | | +80° 25′ | + 86 | | | | ± 3 |
| 6 | Br. 256 | 6·9 | Ko | 2 1 25 | + 54 | − 52 | 28 | + 3 | ± 8 |
| | 82° 51 | 7·1 | | +83° 6′ | + 20 | | | | ± 4 |
| 7 | $\gamma$ Cam | 4·7 | Ao | 3 39 48 | + 29 | − 36 | 28 | − 7 | ± 7 |
| | 70° 259 | 4·7 | | +71° 1′ | + 5 | | | | ± 5 |
| 8 | 66° 406 | 7·8 | Ko | 5 33 41 | + 55 | − 75 | 28 | − 7 | ± 7 |
| | | 7·7 | | +66° 35′ | + 46 | | | | ± 4 |
| 9 | O.A. 7106 | 8·3 | Ko | 6 36 20 | − 11 | − 49 | 22 | − 6 | ± 5 |
| | 70° 419 | 8·9 | | +70° 20′ | − 15 | | | | ± 3 |
| 10 | Br. 1446 | 5·0 | G5 | 10 26 36 | − 11 | − 16 | 23 | + 11 | ± 6 |
| | 76° 393 | 5·8 | | +76° 14′ | − 35 | | | | ± 4 |
| 11 | Ross 451 | ... | Mo | 11 34 38 | ... | ... | 37 | + 36 | ± 10 |
| | | 13·5 | | +67° 53′ | +248 | | | | ± 3 |
| 12 | SW Dra | 10·0 | F4 | 12 12 48 | − 40 | − 9 | 25 | 0 | ± 7 |
| | A.C. 70° 5225 | 10·5 | | +70° 4′ | − 1 | | | | ± 2 |
| 13 | L.F. 2448 | 8·2 | F2 | 14 16 6 | + 27 | + 9 | 27 | + 9 | ± 8 |
| | 69° 743 | 8·2 | | +69° 42′ | + 34 | | | | ± 3 |
| 14 | O.A. 15259 | 8·3 | Ko | 15 12 37 | + 2 | − 51 | 26 | + 12 | ± 8 |
| | 67° 875 | 8·8 | | +67° 14′ | + 16 | | | | ± 3 |

| No. | Name and B.D. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. 1900·0 | Unit 0″·001 Proper Motion R.A. | Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 15 | 11 UMi | 5·1 | Ko | 15 17 10 | + 12 | + 2 | 33 | + 24 | ± 10 |
| | 72° 678 | 6·4 | | +72° 11 | − 5 | | | | ± 4 |
| 16 | L.F. 2757 | 7·6 | Ko | 16 7 29 | − 4 | + 56 | 25 | + 46 | ± 7 |
| | 81° 541 | 8·3 | | +80° 54′ | − 17 | | | | ± 3 |
| 17 | L. 29841 | 8·1 | Ko | 16 14 19 | − 14 | + 58 | 21 | − 7 | ± 6 |
| | 65° 1107 | 8·7 | | +65° 19′ | − 34 | | | | ± 2 |
| 18 | L. 32371 | 8·2 | F2 | 17 31 21 | − 1 | + 11 | 24 | + 19 | ± 9 |
| | 74° 713 | 8·2 | | +74° 34′ | − 1 | | | | ± 6 |
| 19 | O.A. 17779–80 | 9·3 | Ko | 17 57 41 | + 25 | + 70 | 25 | − 2 | ± 8 |
| | 68° 967 | 9·4 | | +68° 50′ | + 8 | | | | ± 3 |
| 20 | 37 Dra | 6·1 | Ko | 18 15 52 | + 15 | − 40 | 20 | − 6 | ± 8 |
| | 68° 984 | 6·8 | | +68° 43′ | + 71 | | | | ± 4 |
| 21 | χ Dra | 3·7 | F8 | 18 22 51 | +496 | −353 | 26 | +126 | ± 10 |
| | 72° 839 | 4·3 | | +72° 41′ | +516 | −386 | | | ± 3 |
| 22 | A.C. 72° 8119 | ... | | 18 59 28 | − 91 | +352 | 31 | − 31 | ± 10 |
| | | 11·5 | | +72° 41′ | − 83 | | | | ± 3 |
| 23 | 80° 606 | 8·3 | Ko | 19 13 37 | − 19 | +124 | 26 | − 8 | ± 9 |
| | | 8·7 | | +80° 21′ | − 24 | | | | ± 4 |
| 24 | Gr. 3478 | 6·9 | Go | 21 23 26 | −120 | +142 | 23 | + 14 | ± 7 |
| | 75° 787 | 7·1 | | +76° 7′ | −142 | | | | ± 3 |
| 25 | 64° 1574 | 8·3 | Ko | 21 35 41 | + 14 | + 8 | 26 | + 11 | ± 9 |
| | | 8·8 | | +65° 12′ | − 3 | | | | ± 4 |
| 26 | 78 Dra | 5·4 | Ko | 21 41 51 | − 44 | − 42 | 29 | + 12 | ± 9 |
| | 71° 1082 | 6·1 | | +71° 52′ | − 61 | | | | ± 4 |
| 27 | 66° 1454 | 9·6 | | 21 51 45 | +244 | + 98 | 27 | + 27 | ± 9 |
| | | 10·4 | | +66° 15′ | +237 | | | | ± 4 |
| 28 | 69° 1216 | ... | | 22 2 11 | − 17 | − 10 | 30 | + 20 | ± 10 |
| | | 10·6 | | +69° 43′ | + 4 | | | | ± 5 |
| 29 | L.F. 4022 | 8·4 | Ko | 22 2 11 | + 64 | + 34 | 29 | − 5 | ± 8 |
| | 69° 1216 | 8·9 | | +69° 44′ | + 72 | | | | ± 4 |
| 30 | O.A. 23583 | 8·6 | Ko | 22 6 14 | + 91 | − 5 | 31 | 0 | ± 10 |
| | 68° 1278 | 8·7 | | +69° 4′ | + 73 | | | | ± 4 |
| 31 | Gr. 3778 | 8·0 | Ko | 22 21 5 | + 86 | − 4 | 30 | − 4 | ± 8 |
| | 74° 963 | 8·2 | | +74° 30′ | + 53 | | | | ± 4 |
| 32 | O.A. 24565 | 8·8 | Ko | 22 37 59 | +102 | + 11 | 23 | + 4 | ± 6 |
| | 73° 987 | 9·1 | | +73° 52′ | + 92 | | | | ± 2 |
| 33 | 77° 876 | 10·3 | | 22 46 55 | +248 | − 15 | 28 | + 3 | ± 7 |
| | | 10·4 | | +77° 25′ | +249 | | | | ± 2 |

# STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE. (TENTH LIST.)

(Communicated by H.M. Astronomer)

The form of publication is similar to that of previous lists.

Several pairs of stars are included. Generally either a different exposure time or a different occulting shutter is required for the two stars. In such cases the determinations are really independent and each star measured is numbered. As the parallaxes of the two Boss stars given under 842 were determined from the same exposures on the same set of plates they have been numbered only once. The four stars 879–882 were measured on the same set of parallax plates but have been counted separately as only one of the stars has a large proper motion.

The results for companions to 863 and 869 have been published previously and are repeated here with the original number. The two determinations are independent as different series of plates were used.

For the star 892 no occulting shutter was required, but for 893 a shutter 1·0 was used. The actual difference in photographic magnitude must be at least one magnitude less than that given. It was generally possible to use plates exposed for one of the stars for the other star. The proper motions derived from the parallax plates are discordant. When the means of the measures for the two stars are used the probable error of unit weight is greatly reduced as the sign of the residuals is often different. The results given under mean were derived from the mean of the measures reduced independently.

The parallax of star 886 as determined at the Cape Observatory is $+''\cdot108 \pm ''\cdot007$ as compared with $+''\cdot013 \pm ''\cdot011$ in the Yale General Catalogue from a determination at Johannesburg. The spectroscopic parallax is $''\cdot041$.

There are 8 stars with a parallax exceeding $''\cdot100$, of which two are new. Both the new stars have a proper motion a little less than 1″ a year and were found in the Cape Astrographic Zone $-40°$ to $-52°$.

As in the previous list, Z.C. refers to the Cape Zone Catalogue ($-40°$ to $-52°$) of 20843 stars for 1900. The letters F.S. refer to the Catalogue of Faint Stars for the same zone now in the press. The letters G.C. refer to the General Catalogue of Stars prepared at the Dudley Observatory, Albany, by Professor Benjamin Boss and published by the Carnegie Institution. P.G.C. refers to the Preliminary General Catalogue by Professor Lewis Boss, frequently referred to as Boss. Ci. 18 and Ci. 20 refer to volumes published by Cincinnati Observatory. U.O.C. stands for Union Observatory Circular.

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Unit 0″·001 Proper Motion R.A. | Proper Motion Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 837 | Ci. 20, 14 | 8·6 | G5 | 0 8 56 | + 412 | −190 | 19 | + 34 | ± 6 |
| | −12° 20 | 9·4 | | −11° 51′·8 | + 408 | | | | ± 6 |
| 838 | P.G.C. 139 | 5·79 | Go | 0 35 44 | + 867 | +446 | 19 | + 54 | ± 6 |
| | −60° 46 | 6·3 | | −60° 1′·0 | + 864 | | | | ± 5 |
| 839 | U.O.C. 53 | 9·1 | Go | 1 11 0 | + 162 | −318 | 21 | + 44 | ± 6 |
| | −65° 125 | 9·7 | | −65° 16′·8 | + 163 | | | | ± 6 |
| 840 | Ci. 20, 94 | 8·1 | Ko | 1 13 32 | + 405 | −270 | 18 | + 81 | ± 6 |
| | −1° 167 | 9·1 | | −1° 23′·2 | + 381 | | | | ± 5 |
| 841 | $\theta$ Ceti | 3·83 | Ko | 1 19 1 | − 80 | −213 | 20 | + 27 | ± 7 |
| | P.G.C. 313 | 4·83 | | −8° 42′·0 | − 107 | | | | ± 7 |
| 842 | P.G.C. 376 | 6·03 | G5 | 1 36 0 | + 281 | − 31 | 25 | +138 | ± 6 |
| | −56° 329*p* | 6·7 | | −56° 42′·2 | + 320 | | | | ± 7 |
| | P.G.C. 377 | 6·00 | G5 | 1 36 1 | + 273 | + 75 | 24 | +129 | ± 6 |
| | −56° 329*f* | 6·7 | | −56° 42′·1 | + 288 | | | | ± 7 |
| | Mean | | | | + 277 | | | +133 | ± 5 |
| | | | | | + 304 | | | | ± 6 |
| 843 | U.O.C. 48 | 8·4 | G5 | 1 37 42 | + 340 | 0 | 20 | + 20 | ± 6 |
| | −53° 350 | 9·3 | | −53° 26′·2 | + 118 | | | | ± 4 |
| 844 | Ci. 20, 135 | 6·53 | G5 | 1 52 17 | − 354 | −240 | 28 | + 20 | ± 10 |
| | −10° 403 | 7·31 | | −10° 43′·5 | − 386 | | | | ± 7 |
| 845 | A.J. 979 | 9·1 | K2 | 1 53 12 | + 484 | +300 | 24 | + 51 | ± 7 |
| | −60° 167 | 10·0 | | −60° 43′·2 | + 417 | | | | ± 6 |
| 846 | F.S. 651 | ... | ... | 2 27 27 | + 9 | −309 | 18 | + 61 | ± 6 |
| | −47° 765 | 12 | | −47° 7′·5 | − 39 | | | | ± 4 |
| 847 | P.G.C. 605 | 5·01 | F5 | 2 34 44 | + 151 | −236 | 29 | + 52 | ± 9 |
| | −12° 501 | 5·43 | | −12° 17′·8 | + 111 | | | | ± 8 |
| 848 | Anon. | ... | ... | 2 45 50 | ... | ... | 26 | − 2 | ± 9 |
| | | 11 | | −69° 19′·8 | − 64 | | | | ± 6 |
| 849 | P.G.C. 696 | 4·16 | A3 | 2 57 59 | − 142 | − 47 | 20 | + 69 | ± 6 |
| | −24° 1387 | 4·24 | | −24° 1′·0 | − 186 | | | | ± 4 |
| 850 | F.S. 821 | ... | ... | 2 59 3 | + 187 | +253 | 14 | + 40 | ± 5 |
| | −45° 1000 | 13 | | −45° 2′·1 | + 186 | | | | ± 5 |
| 851 | F.S. 873 | ... | ... | 3 7 51 | + 200 | +400 | 19 | + 72 | ± 6 |
| | −46° 943 | 12·7 | | −46° 54′·3 | + 166 | | | | ± 5 |

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Unit 0″·001 Proper Motion R.A. | Proper Motion Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 852 | ϵ Eridani | 3·81 | K0 | 3 28 13 | − 971 | + 13 | 27 | +296 | ± 8 |
| | P.G.C. 814 | 4·81 | | −9° 47′·8 | −1031 | | | | ± 7 |
| 853 | Ci. 18, 475 | 8·7 | G5 | 3 31 0 | − 133 | −254 | 27 | + 13 | ± 10 |
| | −9° 709 | 9·5 | | −9° 23′·3 | − 76 | | | | ± 7 |
| 854 | 27 Eridani | 4·33 | F8 | 3 42 33 | − 162 | −523 | 30 | + 50 | ± 9 |
| | P.G.C. 873 | 4·83 | | −23° 32′·7 | − 186 | | | | ± 7 |
| 855 | Ci. 18, 534 | 8·7 | K2 | 3 52 25 | − 213 | −165 | 21 | + 99 | ± 7 |
| | −1° 565 | 9·8 | | −1° 26′·6 | − 220 | | | | ± 4 |
| 856 | γ Eridani | 3·19 | K5 | 3 53 22 | + 67 | −112 | 17 | + 8 | ± 6 |
| | P.G.C. 915 | 4·37 | | −13° 47′·6 | + 26 | | | | ± 4 |
| 857 | Ci. 18, 547 | 8·33 | G5 | 3 59 24 | − 237 | −333 | 29 | + 36 | ± 10 |
| | −0° 636 | 9·11 | | −0° 1′·1 | − 237 | | | | ± 8 |
| 858 | F.S. 1198 | ... | ... | 4 3 38 | − 89 | +328 | 24 | + 43 | ± 8 |
| | −41° 1288 | 12·8 | | −41° 1′·7 | − 96 | | | | ± 9 |
| 859 | F.S. 1265 | ... | ... | 4 14 23 | + 118 | +242 | 27 | − 23 | ± 9 |
| | −43° 1361 | 11·5 | | −43° 38′·5 | + 144 | | | | ± 8 |
| 860 | F.S. 1348 | ... | ... | 4 29 15 | + 185 | +168 | 26 | + 25 | ± 8 |
| | −43° 1464 | 11·9 | | −43° 14′·1 | + 167 | | | | ± 7 |
| 861 | F.S. 1583 | ... | K0 | 5 0 55 | + 189 | −124 | 18 | + 49 | ± 6 |
| | −42° 1743 | 10·6 | | −42° 29′·9 | + 192 | | | | ± 4 |
| 862 | Ci. 18, 668 | 8·17 | G0 | 5 5 11 | − 89 | −220 | 19 | + 23 | ± 7 |
| | −7° 989 | 8·73 | | −7° 11′·5 | − 22 | | | | ± 8 |
| 863 | F.S. 1648 | ... | ... | 5 8 46 | + 46 | −190 | 34 | + 12 | ± 10 |
| | −43° 1726*p* | 11·7 | | −43° 0′·0 | + 66 | | | | ± 7 |
| 677 | F.S. 1649 | 10·1 | G | 5 8 47 | + 48 | −191 | 26 | + 12 | ± 9 |
| | −43° 1726*f* | 10·9 | | −42° 59′·8 | + 71 | | | | ± 10 |
| | Mean | | | | + 47 | | | + 12 | ± 7 |
| | | | | | + 68 | | | | ± 6 |
| 864 | Ci. 18, 706 | 8·38 | G5 | 5 26 36 | + 152 | −484 | 28 | + 43 | ± 10 |
| | −0° 981 | 9·16 | | +0° 2′·2 | + 182 | | | | ± 8 |
| 865 | Z.C. 3000 | 7·4 | G0 | 5 34 58 | + 90 | +283 | 25 | + 57 | ± 8 |
| | −43° 1954 | 7·8 | | −43° 1′·7 | + 92 | | | | ± 8 |
| 866 | F.S. 1882 | ... | K7 | 5 44 42 | + 22 | −325 | 30 | + 18 | ± 11 |
| | −48° 1982 | 11·1 | | −48° 33′·0 | + 35 | | | | ± 11 |

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Proper Motion (Unit 0″·001) R.A. | Proper Motion (Unit 0″·001) Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 867 | B.S. 2612 | 6·19 | F5 | 6 53 43 | − 22 | + 3 | 24 | + 66 | ± 8 |
| | −35° 3233 | 6·7 | | −35° 22′·4 | − 17 | | | | ± 9 |
| 868 | P.G.C. 1857 | 5·11 | K2 | 7 8 6 | − 35 | +198 | 19 | + 54 | ± 6 |
| | −48° 2765 | 7·1 | | −48° 46′·3 | − 50 | | | | ± 6 |
| 869 | −34° 3610 | ... | F8 | 7 23 18 | − 266 | + 90 | 23 | + 27 | ± 7 |
| | | 9·3 | | −34° 7′·0 | − 306 | | | | ± 1 |
| 170 | −34° 3611 | 6·75 | F8 | 7 23 19 | − 286 | + 80 | 27 | + 16 | ± 8 |
| | | 7·4 | | −34° 6′·8 | − 257 | | | | ± 8 |
| | Mean | | | | − 276 | + 85 | | + 22 | ± 6 |
| | | | | | − 305 | | | | ± 1 |
| 870 | Z.C. 5481 | 9·0 | K0 | 8 7 29 | + 21 | −342 | 30 | + 17 | ± 10 |
| | −42° 3961 | 9·2 | | −42° 30′·2 | + 19 | | | | ± 8 |
| 871 | F.S. 3950 | 9·60 | K0 | 8 9 22 | + 207 | −153 | 26 | + 39 | ± 8 |
| | −40° 3981 | 9·8 | | −40° 13′·9 | + 220 | | | | ± 6 |
| 872 | G.C. 11724 | 6·15 | A5 | 8 28 46 | − 8 | − 20 | 26 | + 12 | ± 8 |
| | −24° 7089 | 7·2 | | −24° 16′ | − 11 | | | | ± 9 |
| 873 | P.G.C. 2558 | 3·64 | F5 | 9 26 46 | − 188 | + 66 | 20 | + 61 | ± 6 |
| | −39° 5580 | 4·06 | | −40° 1′·7 | − 222 | | | | ± 7 |
| 874 | F.S. 7996 | ... | ... | 10 31 0 | − 180 | + 12 | 25 | + 27 | ± 8 |
| | −42° 6342 | 10·8 | | −42° 41′·5 | − 151 | | | | ± 8 |
| 875 | Z.C. 9019 | 7·8 | G0 | 10 48 59 | − 261 | + 10 | 16 | + 55 | ± 5 |
| | −43° 6619 | 8·4 | | −43° 52′·8 | − 259 | | | | ± 5 |
| 876 | F.S. 8508 | ... | ... | 10 56 27 | − 207 | − 44 | 20 | + 8 | ± 6 |
| | −43° 6702 | 11·1 | | −44° 7′·6 | − 204 | | | | ± 7 |
| 877 | U.O.C. 49 | ... | ... | 11 11 55 | −2506 | +973 | 24 | + 68 | ± 8 |
| | | 12 | | −57° 1′·4 | −2453 | | | | ± 7 |
| 878 | Z.C. 9551 | 7·4 | F2 | 11 25 8 | − 356 | +106 | 21 | + 49 | ± 6 |
| | −50° 6060 | 8·0 | | −51° 6′·8 | − 358 | | | | ± 6 |
| 879 | −51° 5972 | ... | ... | 11 38 15 | ... | ... | 28 | + 17 | ± 8 |
| | | 12 | | −51° 15′·6 | + 16 | | | | ± 9 |
| 880 | F.S. 9267 | ... | ... | 11 38 18 | + 2 | − 4 | 28 | + 21 | ± 8 |
| | −51° 5974*p* | 11·4 | | −51° 16′·2 | + 31 | | | | ± 9 |
| 881 | F.S. 9268 | ... | ... | 11 38 19 | + 679 | −540 | 20 | +117 | ± 6 |
| | −51° 5974*f* | 11·4 | | −51° 16′·2 | + 684 | | | | ± 6 |

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Unit 0″·001 Proper Motion R.A. | Proper Motion Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 882 | F.S. 9270 | ... | ... | 11 38 25 | − 20 | − 7 | 40 | + 20 | ± 12 |
| | −51° 5976 | 10·9 | | −51° 17′·7 | − 42 | | | | ± 13 |
| 883 | G.C. 16731 | 6·12 | F8 | 12 10 2 | + 32 | −1024 | 28 | + 24 | ± 9 |
| | −9° 3468 | 6·62 | | −9° 43′·7 | + 36 | | | | ± 7 |
| 884 | P.G.C. 3272 | 4·42 | Fo | 12 26 55 | − 441 | − 67 | 24 | + 25 | ± 9 |
| | −15° 3489 | 4·70 | | −15° 38′·5 | − 414 | | | | ± 10 |
| 885 | Ci. 20, 757 | 8·7 | G5 | 12 56 10 | − 490 | − 70 | 29 | + 22 | ± 8 |
| | −7° 3525 | 9·5 | | −7° 54′·0 | − 454 | | | | ± 10 |
| 886 | P.G.C. 3422 | 5·09 | F2 | 13 6 44 | + 91 | −300 | 22 | +108 | ± 7 |
| | −15° 3613 | 5·43 | | −15° 39′·6 | + 83 | | | | ± 6 |
| 887 | Ross 484 | ... | ... | 13 13 52 | − 674 | −230 | 19 | + 47 | ± 6 |
| | Ci. 20, 770 | 11·4 | | −2° 32′·5 | − 653 | | | | ± 7 |
| 888 | Wolf 485 | ... | ... | 13 25 5 | −1099 | −420 | 25 | + 71 | ± 7 |
| | −7° 3632 | 12 | | −8° 3′·6 | −1077 | | | | ± 8 |
| 889 | Ci. 20, 806 | ... | ... | 13 42 12 | − 358 | −570 | 26 | + 28 | ± 8 |
| | −5° 3763 | 11·5 | | −5° 37′·7 | − 352 | | | | ± 10 |
| 890 | F.S. 12683 | ... | K2 | 14 56 37 | − 279 | −270 | 28 | + 35 | ± 8 |
| | −42° 9957 | 11·1 | | −42° 12′·8 | − 214 | | | | ± 10 |
| 891 | Ci. 20, 920 | ... | ... | 15 8 51 | − 748 | +200 | 27 | + 37 | ± 8 |
| | −3° 3746 | 11 | | −3° 25′·8 | − 746 | | | | ± 9 |
| 892 | Z.C. 13630 | ... | ... | 15 29 32 | − 78 | −150 | 30 | + 33 | ± 7 |
| | −49° 9800*p* | 11·2 | | −49° 50′·0 | + 8 | | | | ± 8 |
| 893 | Z.C. 13631 | 9·1 | Go | 15 29 32 | − 80 | −137 | 38 | + 16 | ± 9 |
| | −49° 9800*f* | 8·6 | | −49° 49′·9 | − 140 | | | | ± 10 |
| | Mean | | | | − 79 | −144 | 18 | + 24 | ± 4 |
| | | | | | − 67 | | | | ± 5 |
| 894 | Z.C. 14030 | ... | ... | 15 42 21 | − 243 | − 87 | 21 | + 26 | ± 5 |
| | −49° 10033 | 9·9 | | −49° 39′·2 | − 218 | | | | ± 6 |
| 895 | U.O.C. 48 | 7·5 | K2 | 16 1 41 | − 58 | +380 | 24 | + 73 | ± 7 |
| | −56° 7345 | 7·8 | | −56° 11′·2 | − 115 | | | | ± 8 |
| 896 | Ci. 20, 985 | ... | ... | 16 19 21 | − 711 | −140 | 20 | + 63 | ± 5 |
| | −21° 4352 | 11·7 | | −21° 42′·0 | − 652 | | | | ± 7 |
| 897 | U.O.C. 48 | ... | ... | 16 48 0 | − 169 | −311 | 21 | + 25 | ± 6 |
| | ... | 11 | | −57° 40′·8 | − 193 | | | | ± 7 |

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Unit 0″·001 Proper Motion R.A. | Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 898 | Ci. 20, 1024 | 6·06 | F8 | 17 8 46 | − 176 | −410 | 21 | + 20 | ± 5 |
| | −38° 11686 | 6·8 | | −38° 28′·0 | − 218 | | | | ± 4 |
| 899 | P.G.C. 4394 | 4·46 | F5 | 17 15 1 | + 239 | −207 | 19 | + 58 | ± 5 |
| | −20° 4731 | 4·88 | | −21° 0′·3 | + 232 | | | | ± 6 |
| 900 | F.S. 17308 | 10·1 | Ko | 17 40 4 | − 73 | −209 | 25 | + 19 | ± 8 |
| | −48° 12029 | 10·3 | | −48° 48′·8 | − 63 | | | | ± 10 |
| 901 | −30° 14959 | 7·5 | K5 | 17 50 1 | ... | ... | 25 | + 10 | ± 7 |
| | | 8·6 | | −30° 10′·8 | − 5 | | | | ± 7 |
| 902 | Z.C. 17017 | 8·4 | F8 | 17 58 49 | − 72 | −185 | 17 | + 22 | ± 4 |
| | −47° 12051 | 9·2 | | −47° 22′·0 | − 90 | | | | ± 5 |
| 903 | F.S. 17808 | ... | ... | 18 2 47 | − 130 | −206 | 13 | + 13 | ± 3 |
| | −50° 11756 | 11·1 | | −50° 21′·5 | − 148 | | | | ± 4 |
| 904 | Z.C. 17258 | 6·95 | F8 | 18 12 18 | − 84 | −137 | 19 | + 41 | ± 5 |
| | −45° 12390 | 7·5 | | −45° 44′·1 | − 123 | | | | ± 6 |
| 905 | Z.C. 17456 | 9·1 | G5 | 18 24 13 | − 58 | −203 | 18 | + 11 | ± 5 |
| | −43° 12591*p* | 9·2 | | −43° 25′·7 | − 123 | | | | ± 6 |
| 906 | Z.C. 17457 | ... | ... | 18 24 13 | − 65 | −211 | 17 | + 13 | ± 4 |
| | −43° 12591*f* | 11·7 | | −43° 25′·5 | − 38 | | | | ± 6 |
| | Mean | | | | − 62 | −207 | | + 12 | ± 3 |
| | | | | | − 80 | | | | ± 4 |
| 907 | Ci. 20, 1101 | ... | ... | 18 35 19 | − 57 | −657 | 28 | + 31 | ± 10 |
| | −13° 5069 | 11·5 | | −13° 27′·0 | − 91 | | | | ± 11 |
| 908 | U.O.C. 48 | ... | ... | 19 4 11 | − 151 | −390 | 21 | + 52 | ± 6 |
| | Co. D. −56° 7638 | 12 | | −56° 2′·4 | − 259 | | | | ± 6 |
| 909 | F.S. 18710 | ... | ... | 19 25 22 | − 346 | −293 | 30 | + 22 | ± 8 |
| | −42° 14278 | 10·4 | | −42° 24′·9 | − 360 | | | | ± 10 |
| 910 | F.S. 18838 | 10·3 | Go | 19 39 18 | + 38 | −335 | 24 | + 7 | ± 7 |
| | −48° 13255 | 10·5 | | −48° 45′·5 | + 36 | | | | ± 8 |
| 911 | U.O.C. 53 | ... | ... | 19 54 36 | + 89 | −849 | 15 | + 83 | ± 5 |
| | Ci. 20, 1178*p* | 13·5 | | −65° 51′6 | + 114 | | | | ± 4 |
| 912 | U.O.C. 53 | ... | ... | 19 54 39 | + 89 | −849 | 20 | + 61 | ± 5 |
| | Ci. 20, 1178*f* | 12 | | −65° 51′·7 | + 103 | | | | ± 4 |
| | Mean | | | | + 89 | | | + 72 | ± 4 |
| | | | | | + 108 | | | | ± 3 |

| No. | Name and D.M. No. | Mag. Vis. Phot. | Spec. Type | R.A., Dec. (1900) | Unit 0″·001 Proper Motion R.A. | Proper Motion Dec. | P.E. Unit Weight | Parallax | P.E. |
|---|---|---|---|---|---|---|---|---|---|
| | | | | h m s | | | | | |
| 913 | U.O.C. 53 | 8·7 | G5 | 19 55 30 | + 192 | −265 | 24 | + 14 | ± 6 |
| | −65° 3839 | 9·5 | | −65° 38′·0 | + 230 | | | | ± 8 |
| 914 | δ Pavonis | 3·64 | G5 | 19 58 55 | +1154 | −1146 | 27 | +169 | ± 8 |
| | P.G.C. 5138 | 4·42 | | −66° 26′·2 | +1203 | | | | ± 9 |
| 915 | Wolf 896 | ... | ... | 20 51 22 | − 198 | −1122 | 23 | + 51 | ± 7 |
| | Ci. 20, 1242 | 11·5 | | −10° 48′·4 | − 61 | | | | ± 6 |
| 916 | P.G.C. 5601 | 5·70 | G5 | 21 41 46 | + 145 | −307 | 17 | + 68 | ± 5 |
| | −47° 13928 | 7·2 | | −47° 45′·5 | + 123 | | | | ± 5 |
| 917 | U.O.C. 48 | ... | ... | 22 16 52 | + 603 | +230 | 18 | + 57 | ± 6 |
| | Co. D. −57° 8545 | 11·8 | | −57° 42′·5 | + 549 | | | | ± 7 |
| 918 | P.G.C. 5835 | 5·33 | Ko | 22 32 35 | − 78 | −115 | 26 | + 17 | ± 8 |
| | −4° 5716 | 6·33 | | −4° 44′·6 | − 111 | | | | ± 7 |
| 919 | Alb. 7913 | 7·76 | K2 | 22 35 6 | + 327 | + 21 | 24 | + 66 | ± 6 |
| | −30° 19255 | 9·0 | | −30° 11′·7 | + 350 | | | | ± 4 |
| 920 | α Piscis Aust. | 1·29 | A3 | 22 52 8 | + 324 | −166 | 17 | +157 | ± 4 |
| | −30° 19370 | 1·37 | | −30° 9′·1 | + 301 | | | | ± 4 |
| 921 | P.G.C. 6092 | 5·66 | K5 | 23 38 42 | + 4 | + 26 | 26 | + 4 | ± 8 |
| | −65° 4159 | 6·9 | | −64° 57′·6 | − 8 | | | | ± 7 |
| 922 | F.S. 20541 | ... | ... | 23 57 0 | + 618 | −678 | 23 | +135 | ± 8 |
| | U.O.C. 58 | 12·8 | | −43° 41′·7 | + 567 | | | | ± 9 |

*Errata in Previous Lists.*

*M.N.*, 97, 81. No. 678, Parallax should read +″·063 (not +″·022).

,, 98, 86. No. 810, R.A. and Dec. should read $18^h\ 10^m\ 10^s$, −27° 57′·6.

,, 98, 87. No. 826, Observed Proper Motion in R.A., *for* −253 *read* +253.

No. 832, Observed Proper Motion in R.A., *for* +304 *read* +457.